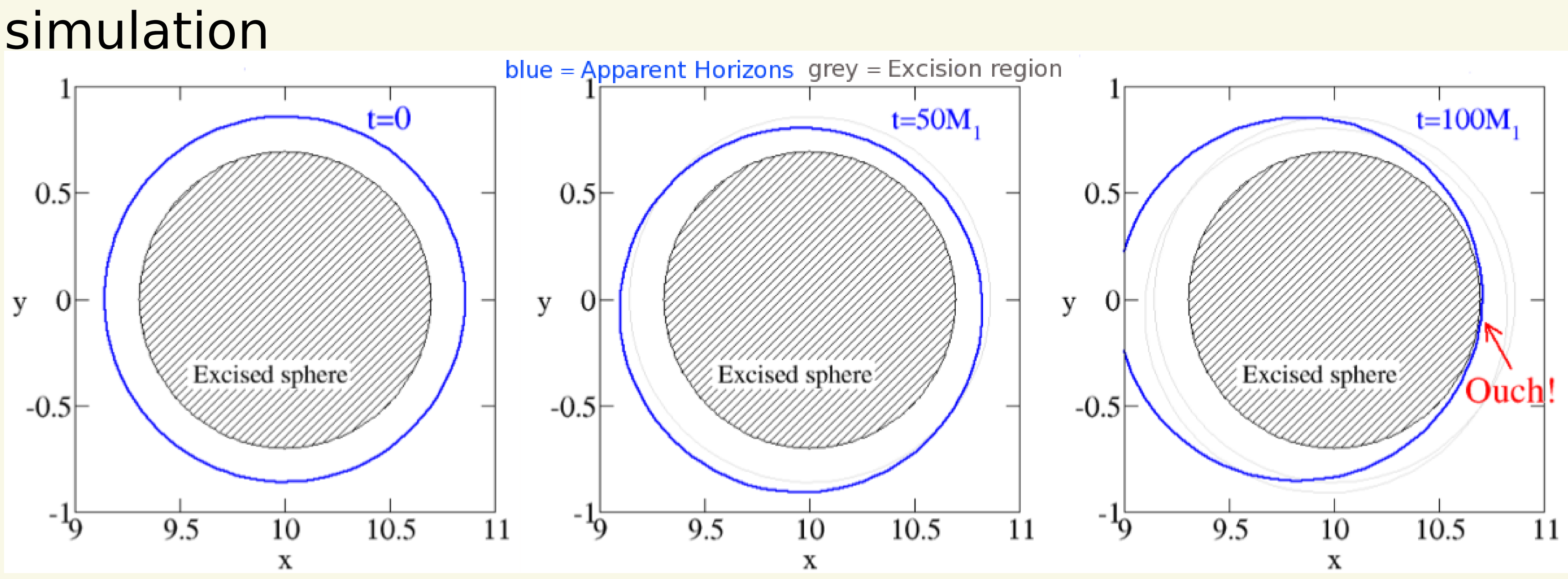
Simulating binary black holes using precession-tracking coordinates

Introduction

- Binary black holes promise to be among the most prominent sources of gravitational waves to be detected by AdvLIGO
- We wish to study the behaviour of *extremely precessing* binaries to investigate new effects in dynamics and in the gravitational wave signal

Simulating precessing binaries with SpEC

- We employ the numerical code SpEC (Spectral Einstein Code) which uses excision to prevent the singularities inside the black holes from causing numerical problems.
- Computational challenge: In an inertial coordinate system the black holes orbit a common centre of gravity; when the black holes move, the apparent horizons of the black holes hit the fixed excision sphere boundaries. No meaningful boundary conditions can then be provided and we must stop the simulation



- Solution: Use coordinates where the apparent horizons do not move. These are related to inertial coordinates by a dynamically-controlled transformation: a rotation, a scaling and a translation
- A control system adjusts the mapping parameters $\{\lambda^\mu\}$ for the next time step by using their derivatives and examining the behaviour of control functions $\{Q^\mu\}$ which are identically zero for perfect control

The challenge

- Need non-singular and well-behaved mapping parameters
- Transformation between the two coordinate frames involves a rotation parametrized by two Euler angles: Pitch and Yaw
- The dual-frame approach relies on the derivatives of the mapping parameters to dynamically adjust them at the next time step
- But the derivatives of Euler angles grow rapidly as the orbit becomes more polar and actually diverge at the poles, so *the simulations break down for highly-precessing binaries*

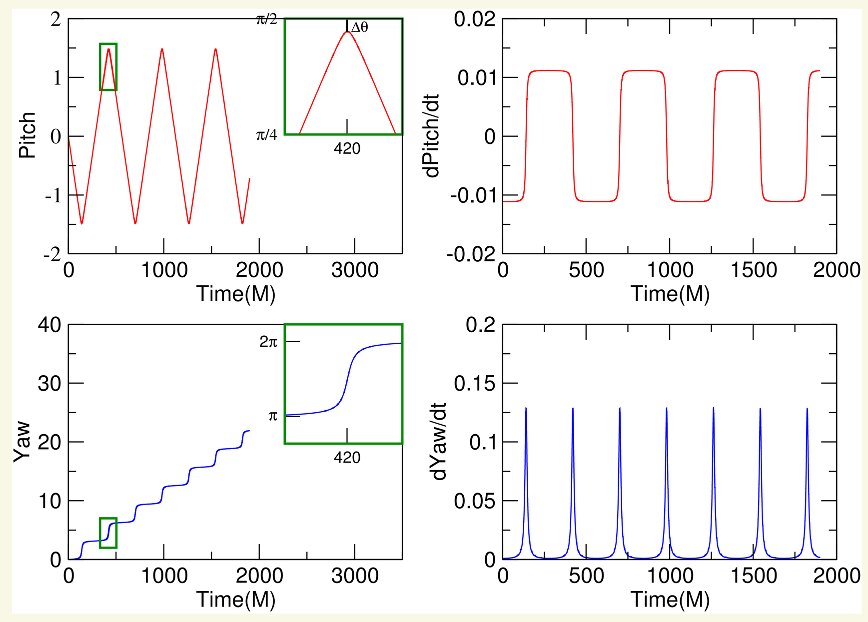
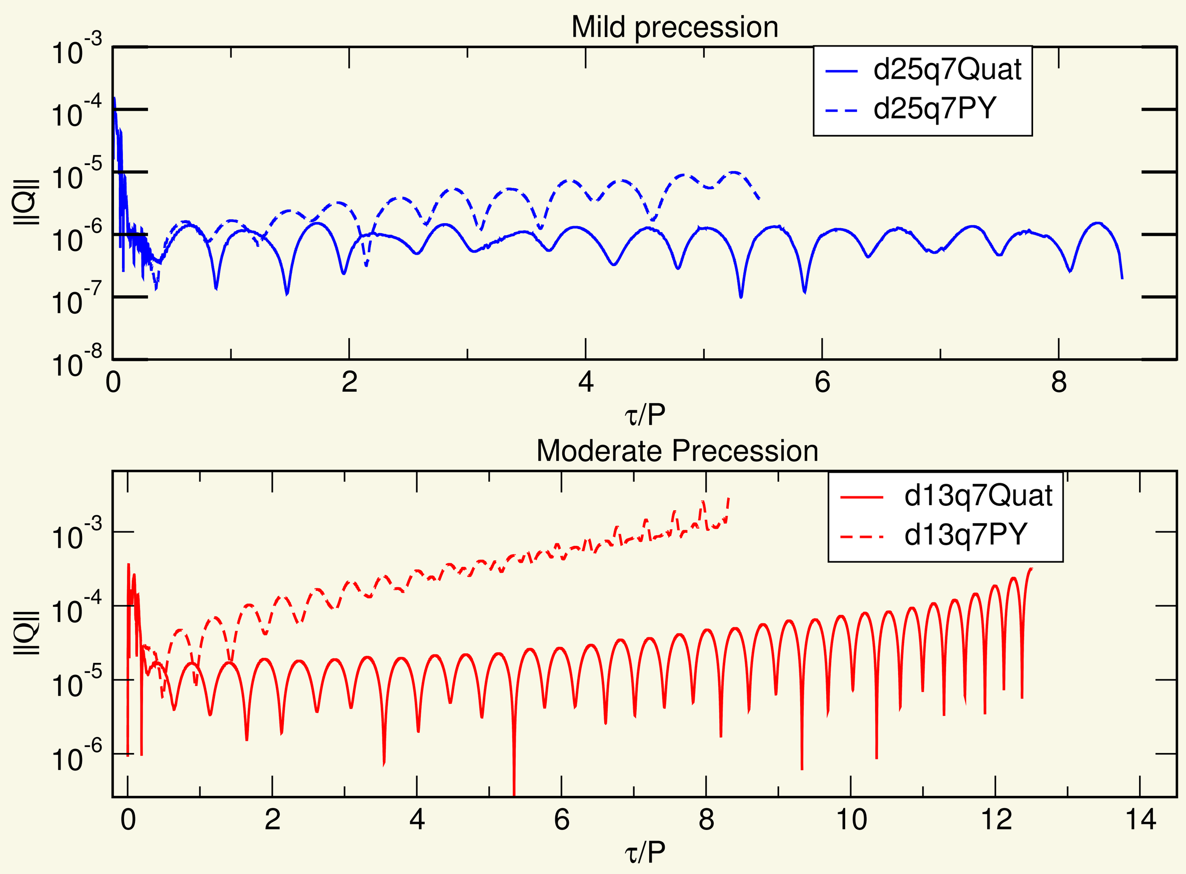


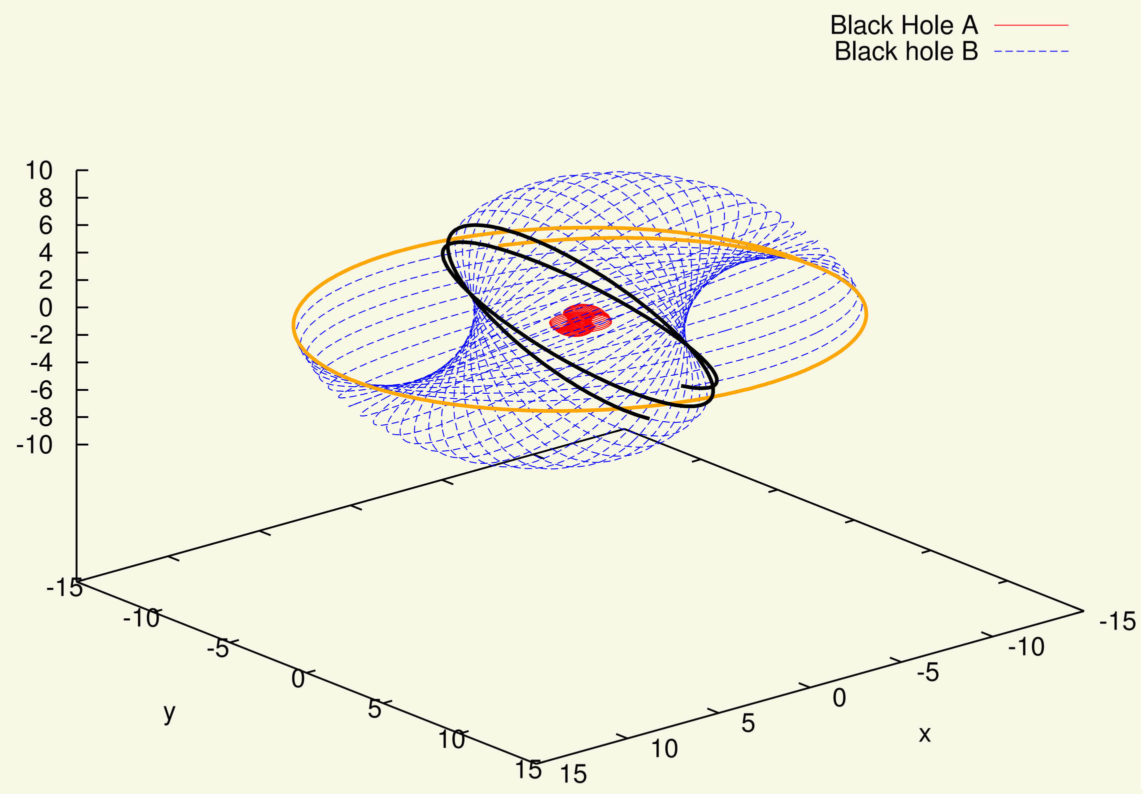
Figure: Behaviour of Euler angles and derivatives for an orbit inclined at 85 with respect to the $x-y$ plane

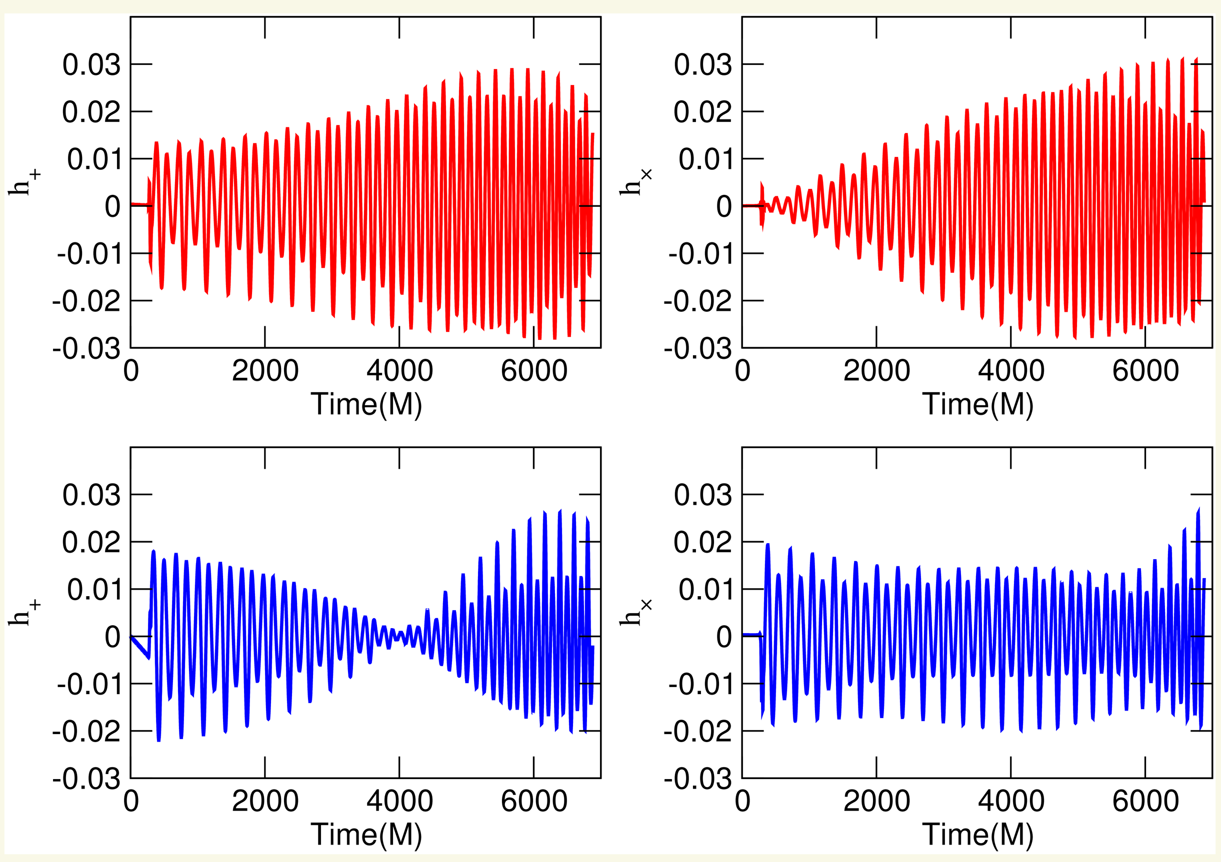
The solution

- Solution: Use *quaternions* to parametrize rotations!
- Quaternions are a generalization of complex numbers by introducing three imaginary units i, j, k.
- They do not rely on rotations around a fixed set of axes, but rather rotations around a single axis and the angle of rotation; thus, they do not have the same problems as Euler's angles. A rotation of a vector $\vec{v} \in \mathbb{R}^3$ by a quaternion q is given by $v' = qvq^*$ where $v = (0, \vec{v})$
- We construct the overall rotation from a sequence of infinitesimal rotations by evolving the ODE $\frac{dq}{dt} = \frac{1}{2}q\Omega$

Numerical results and conclusion







We test the new control system with three full numerical relativity simulations displaying precession of 11 and 76 degrees (above left). Here, $||Q|| = 0$ for ideal control. From the figure above, the Euler angle control system (labelled PY) behaves well for mild precession but then breaks down for high precession. Meanwhile, the quaternion control system (labelled Quat) behaves well in both cases. Armed with the control system we were now able to evolve a run that precesses by 140° (above center, orange - initial orbital plane, black - final). The resulting (preliminary) waveforms can be seen above right. Thus we can conclude

We have created a control system that allows us to investigate arbitrarily precessing black hole binaries

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