

*Numerical Studies of the Hydrodynamic, Spectral  
and Timing Properties of a Two-Component  
Accretion Flow around a Black Hole*

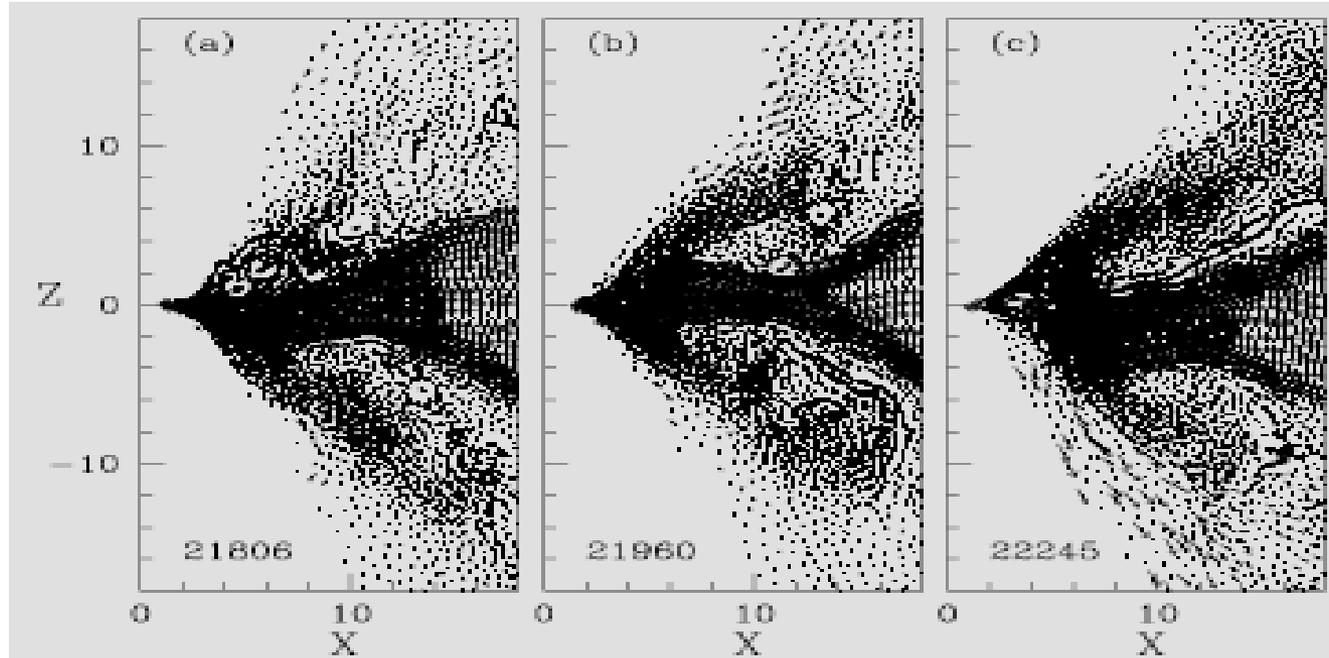
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# Motivation

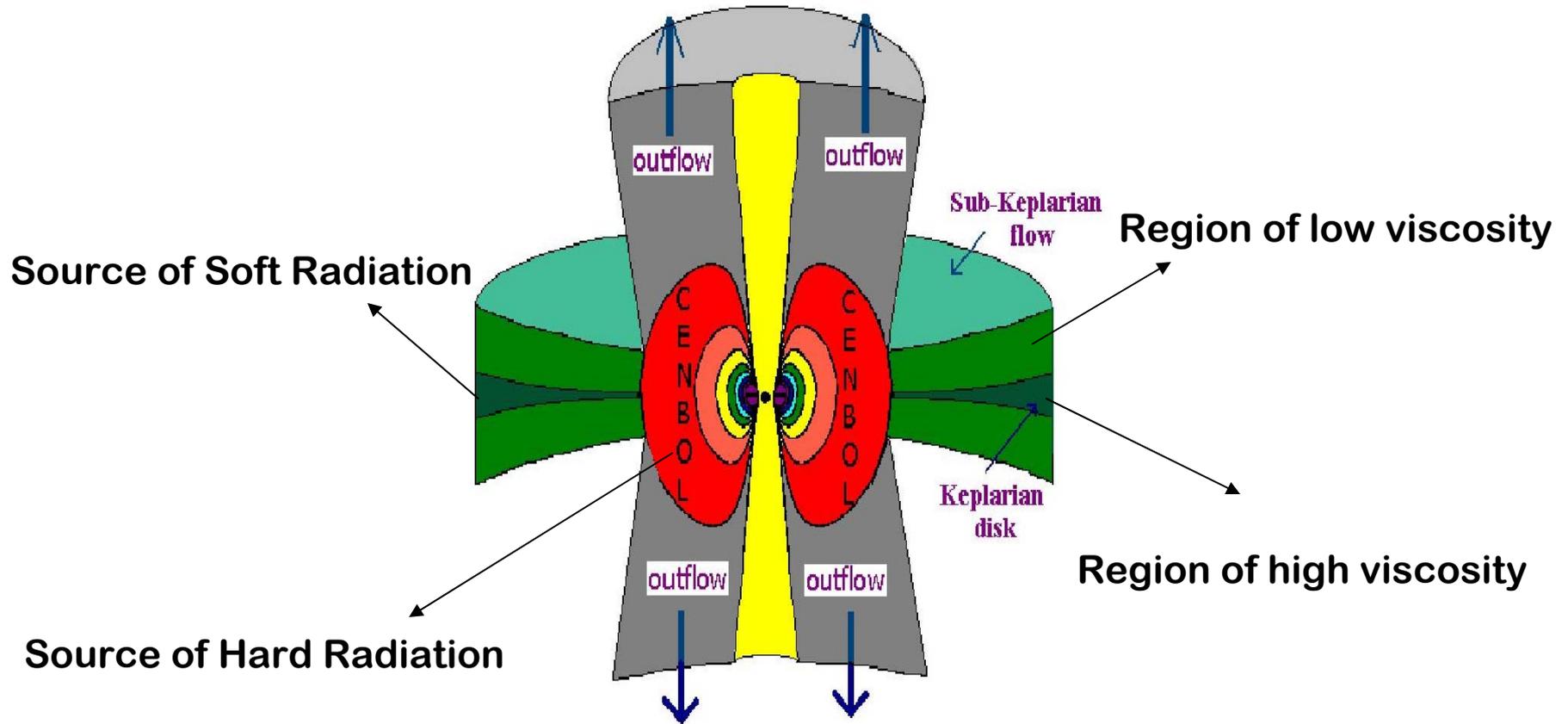


Configuration of the advective flow with cooling effects at three different phases of oscillation when both the radial and the vertical motions of the shocks are allowed. The modulation of X-rays due to this oscillation causes quasi-period oscillations in black hole candidates. The FFT of the time varying luminosity emitted from the disk produces peaks in the power density spectrum very similar to what is observed. **Our goal would be to carry out the computation of the spectrum while the disk oscillates and see if the spectrum ‘rocks’ during the shock oscillations.**

Chakrabarti et al. Proceedings of MG11, 2007.

Chakrabarti, Acharyya, Molteni, A&A, 2004.

# Cartoon Diagram of a General Model



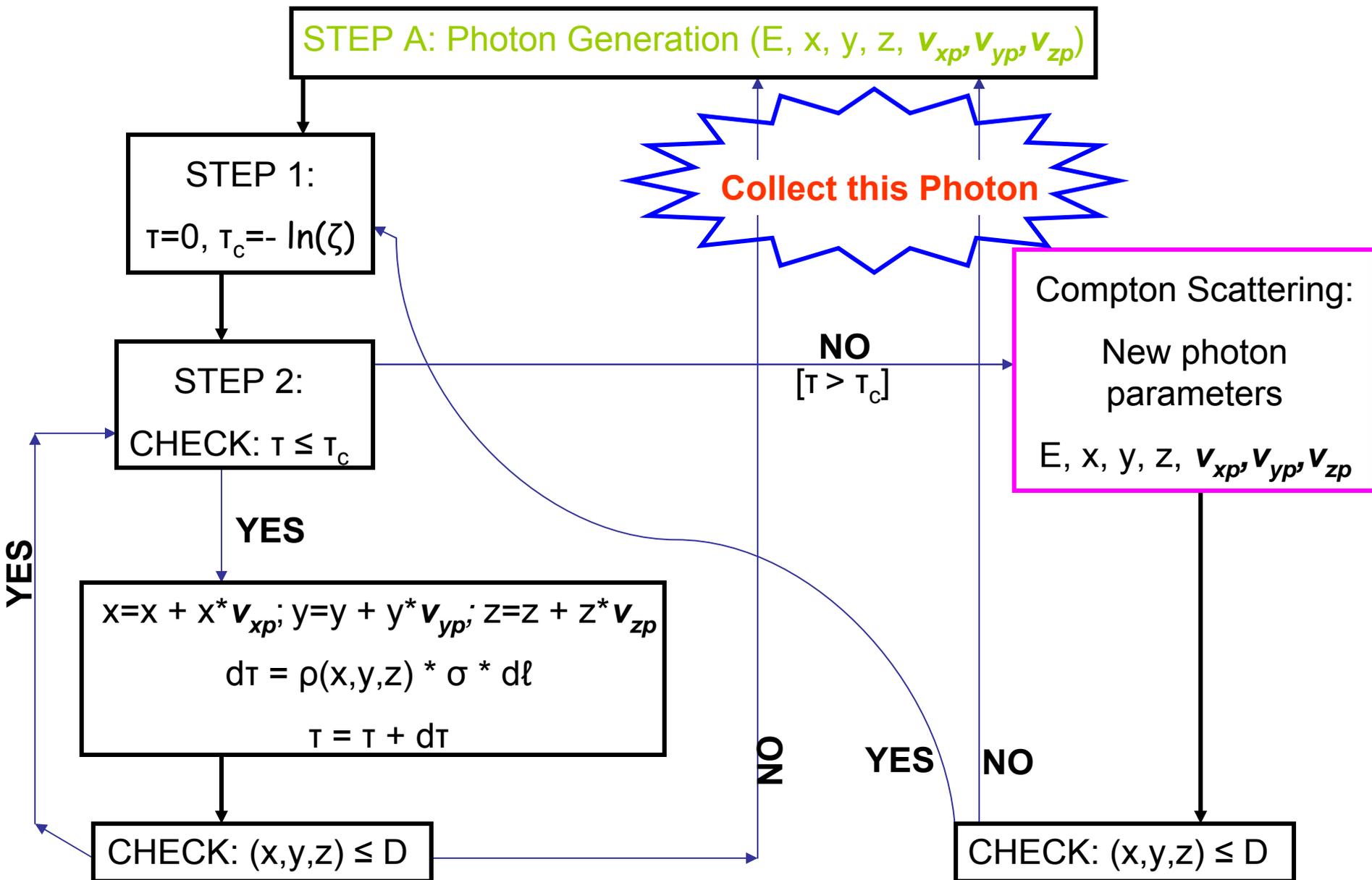
CENBOL: **CEN**trifugal barrier supported **BO**undary **L**ayer

Chakrabarti, Ghosh, Som, 2007

Chakrabarti, Titarchuk, 1995

## *Simulation Technique: Radiative Transfer Code*

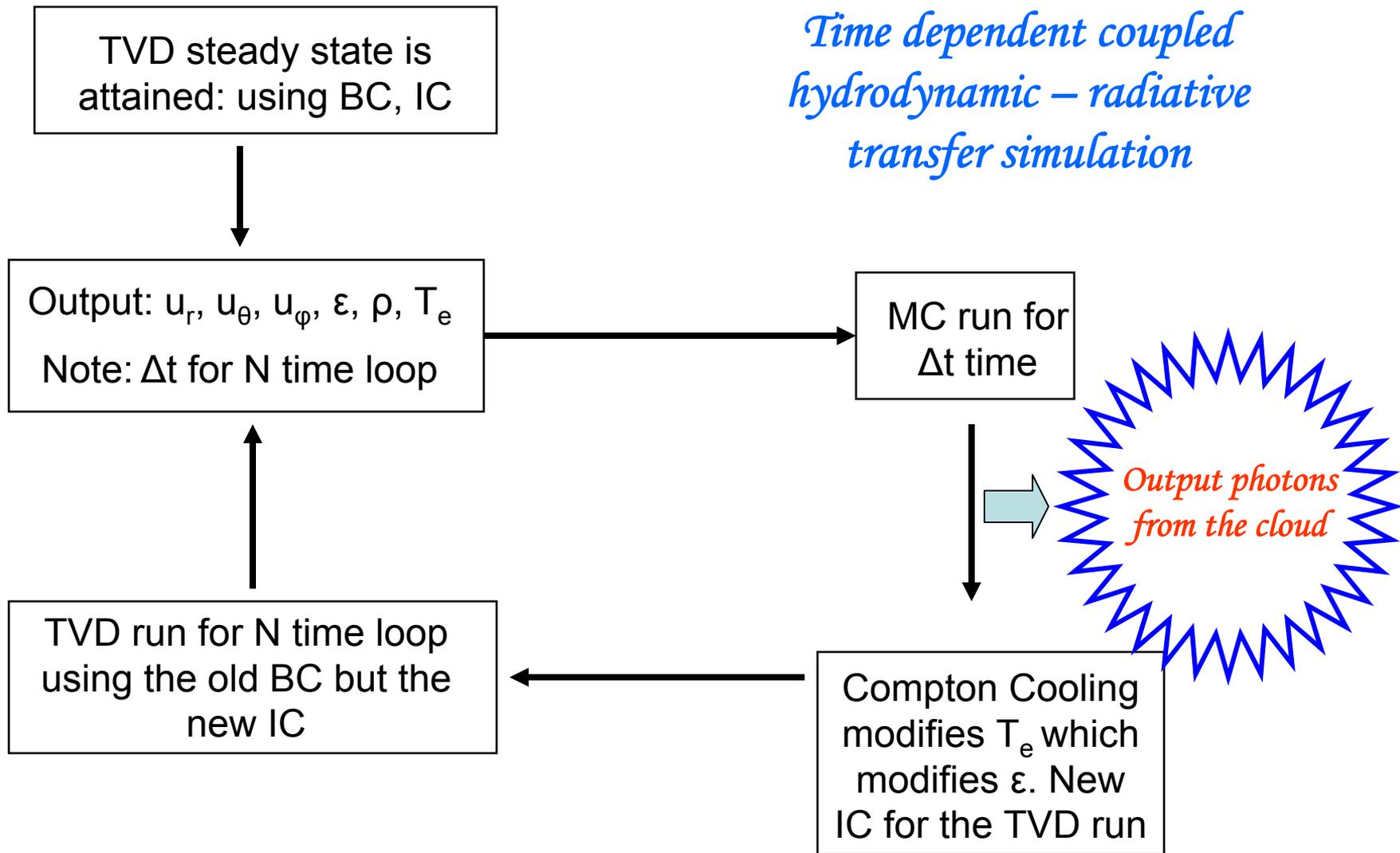
Given:  $T_e$ ,  $\rho_e$ ,  $v_e$ , GEOMETRIC DIMENSION ( $D$ ) OF THE SYSTEM



## **Coupled Hydrodynamic – Radiative Transfer Simulation**

Assuming axisymmetry, we have calculated the flow dynamics using a finite difference method which uses the principle of Total Variation Diminishing (TVD) to carry out hydrodynamic simulations

*Time dependent coupled hydrodynamic – radiative transfer simulation*



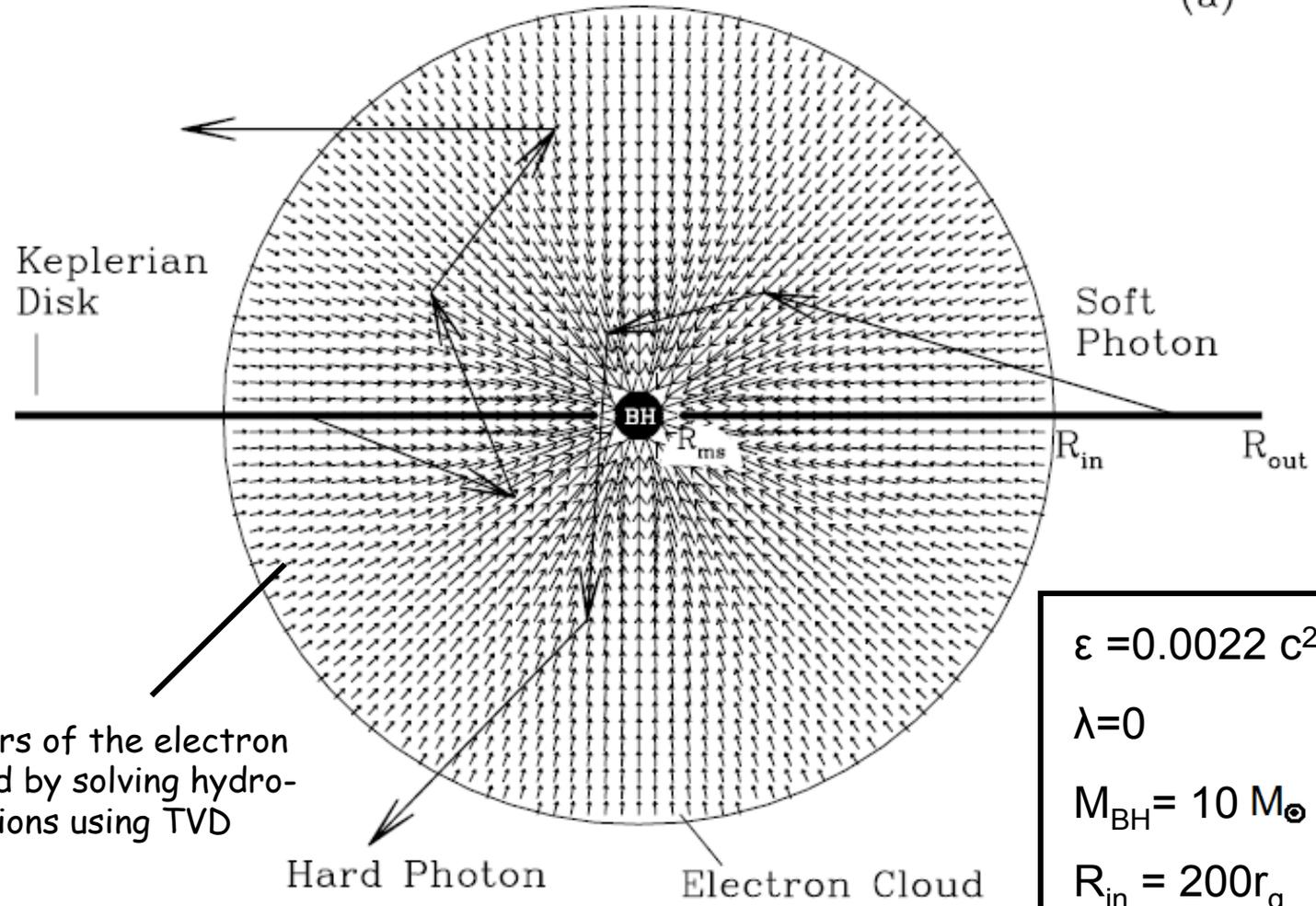
N=100 is used in our simulation

Boundary Condition (BC):  $u_r, u_\theta, u_\phi, \epsilon, \rho$

Initial Condition (IC):  $u_r, u_\theta, u_\phi, \epsilon, \rho, T_e$

# Compton cloud with ZERO angular momentum (Bondi flow)

(a)

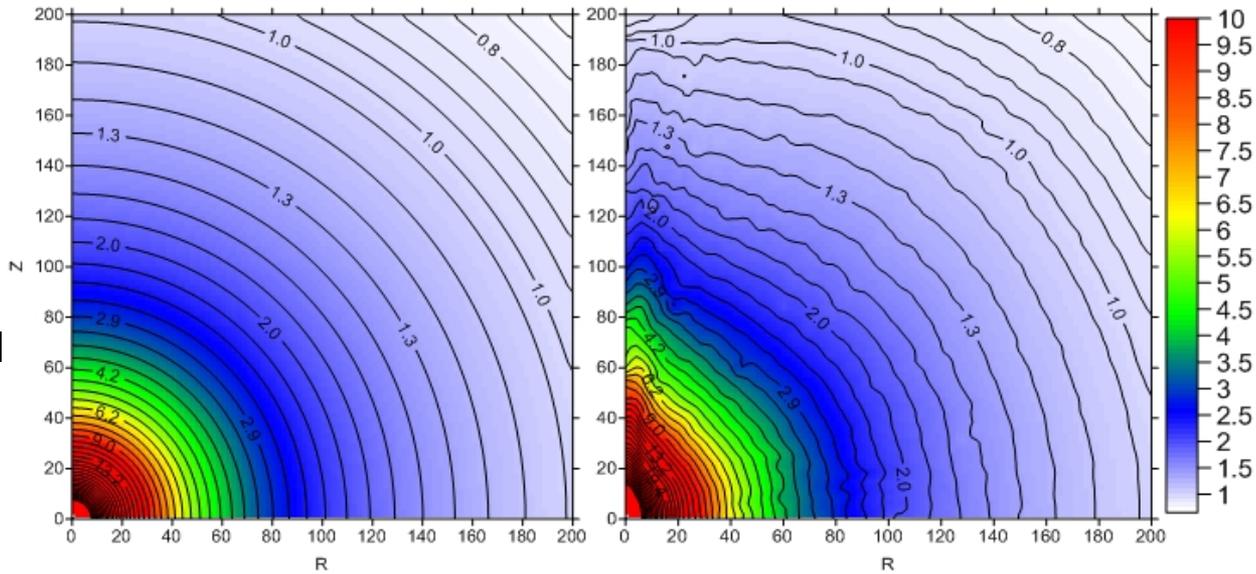


Velocity vectors of the electron cloud obtained by solving hydrodynamic equations using TVD scheme

$\epsilon = 0.0022 c^2$   
 $\lambda = 0$   
 $M_{BH} = 10 M_{\odot}$   
 $R_{in} = 200 r_g$   
 $R_{out} = 300 r_g$   
 $R_{ms} = 3.1 r_g$

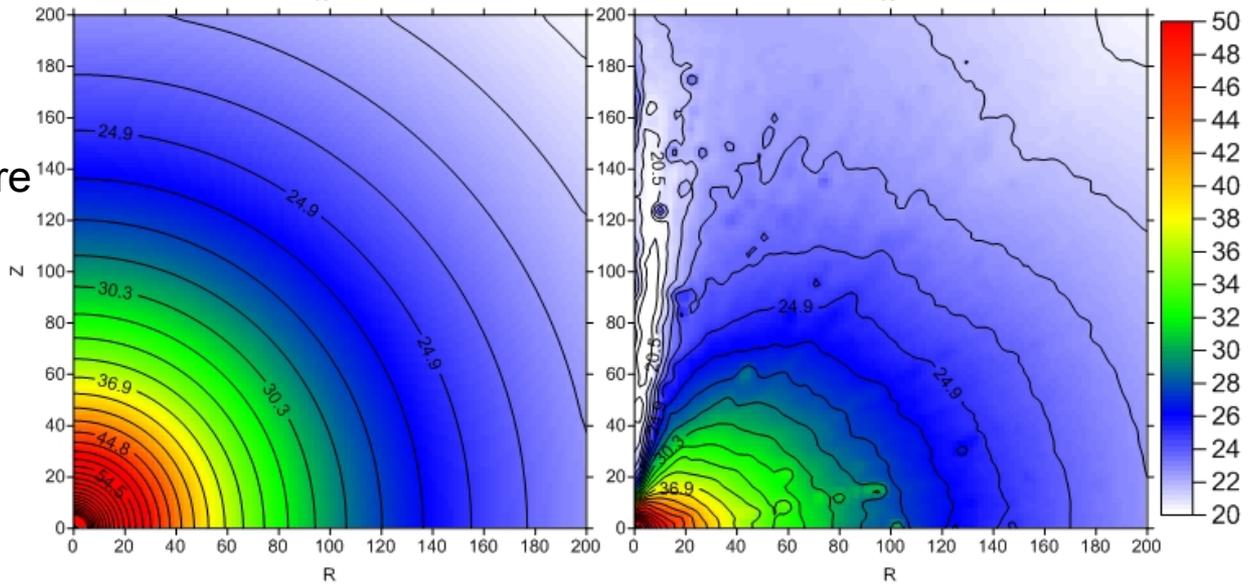
# Density & Temperature contours inside the electron cloud

Initial (no cooling)



Density  
(normalized unit)

Final  
 $\dot{M}_h = 1, \dot{M}_d = 10$



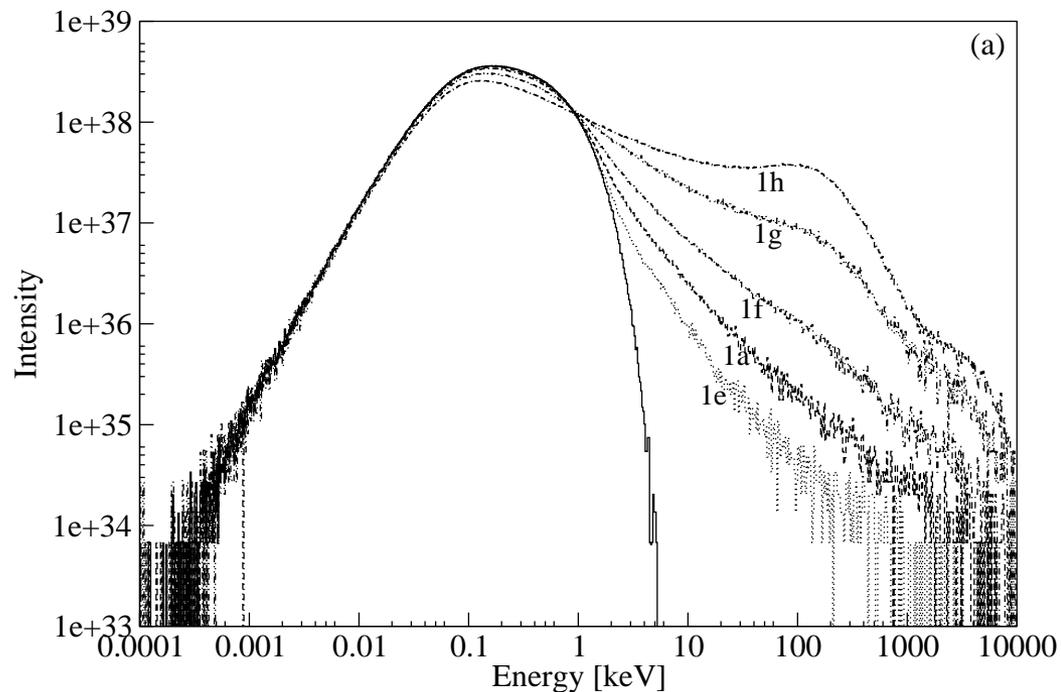
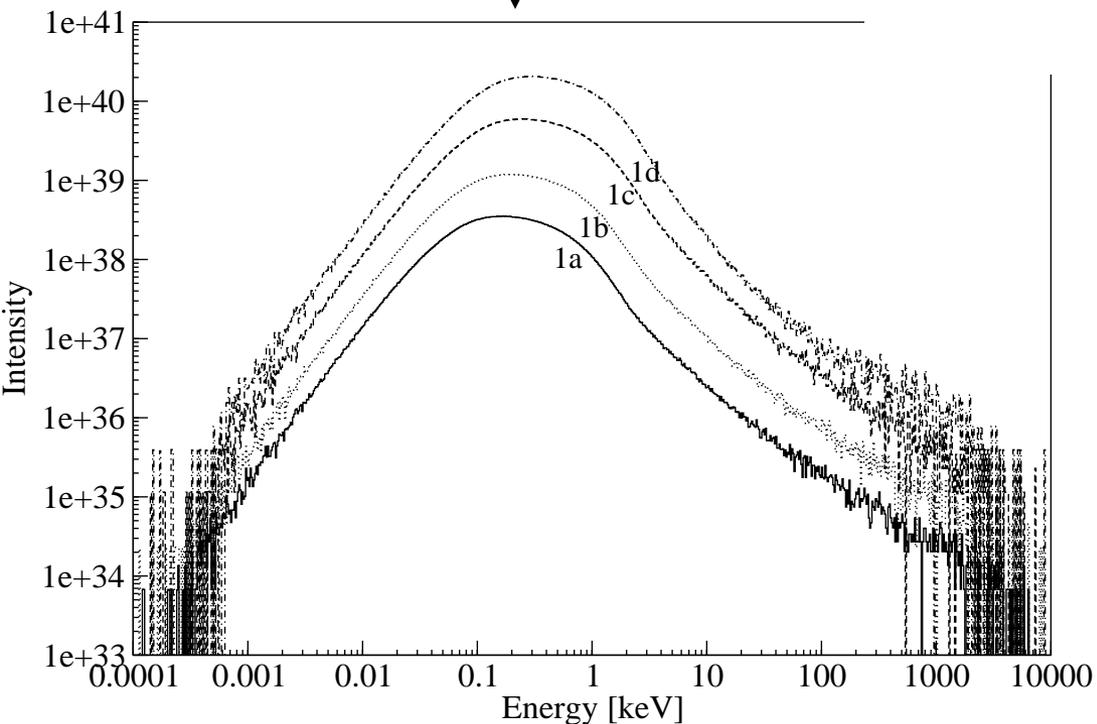
Temperature  
(keV)

## Summary of the simulation cases presented

Case	$\lambda$	$\epsilon$	$\dot{m}_d$	$\dot{m}_h$	$N_{inj}$	$N_{sc}$	$N_{unsc}$	$N_{bh}$ [%]	$p$ [%]	$t_0$ [sec]	$\alpha$
1a	0	22E-4	1	1	4.3E+40	8.7E+39	3.5E+40	0.119	20.030	228.3	1.15, 0.99
1b	0	22E-4	2	1	1.5E+41	2.9E+40	1.2E+41	0.120	20.023	63.6	1.30, 1.0
1c	0	22E-4	5	1	7.3E+41	1.5E+41	5.9E+41	0.121	19.942	12.4	1.40, 0.96
1d	0	22E-4	10	1	2.5E+42	5.0E+41	2.0E+42	0.121	19.816	4.2	1.65, 0.90
1e	0	22E-4	1	0.5	4.3E+40	4.7E+39	3.9E+40	0.070	10.886	380.0	1.57
1f	0	22E-4	1	2	4.3E+40	1.5E+40	2.8E+40	0.230	34.324	118.9	1.1
1g	0	22E-4	1	5	4.3E+40	2.6E+40	1.8E+40	0.502	59.012	48.0	0.7
1h	0	22E-4	1	10	4.3E+40	3.3E+40	1.1E+40	0.699	75.523	35.1	0.45

$\lambda$  => Angular momentum of the flow,  $\epsilon$  => Specific energy of the flow, Photon Index  $\alpha$  is calculated from  $I(E) \sim E^{-\alpha}$ ,  $\dot{M}_h$  => Halo rate in the unit of Eddington rate,  $\dot{M}_d$  => Disk rate,  $N_{inj}$  => #of injected photons,  $N_{sc}$  => #of scattered photons,  $N_{unsc}$  => #of unscattered photons,  $N_{bh}$  => #of photons went inside BH horizon,  $p = N_{sc}/N_{inj}$ ,  $t_0 = \text{Cooling time} = E/(\Delta E/\Delta t)$

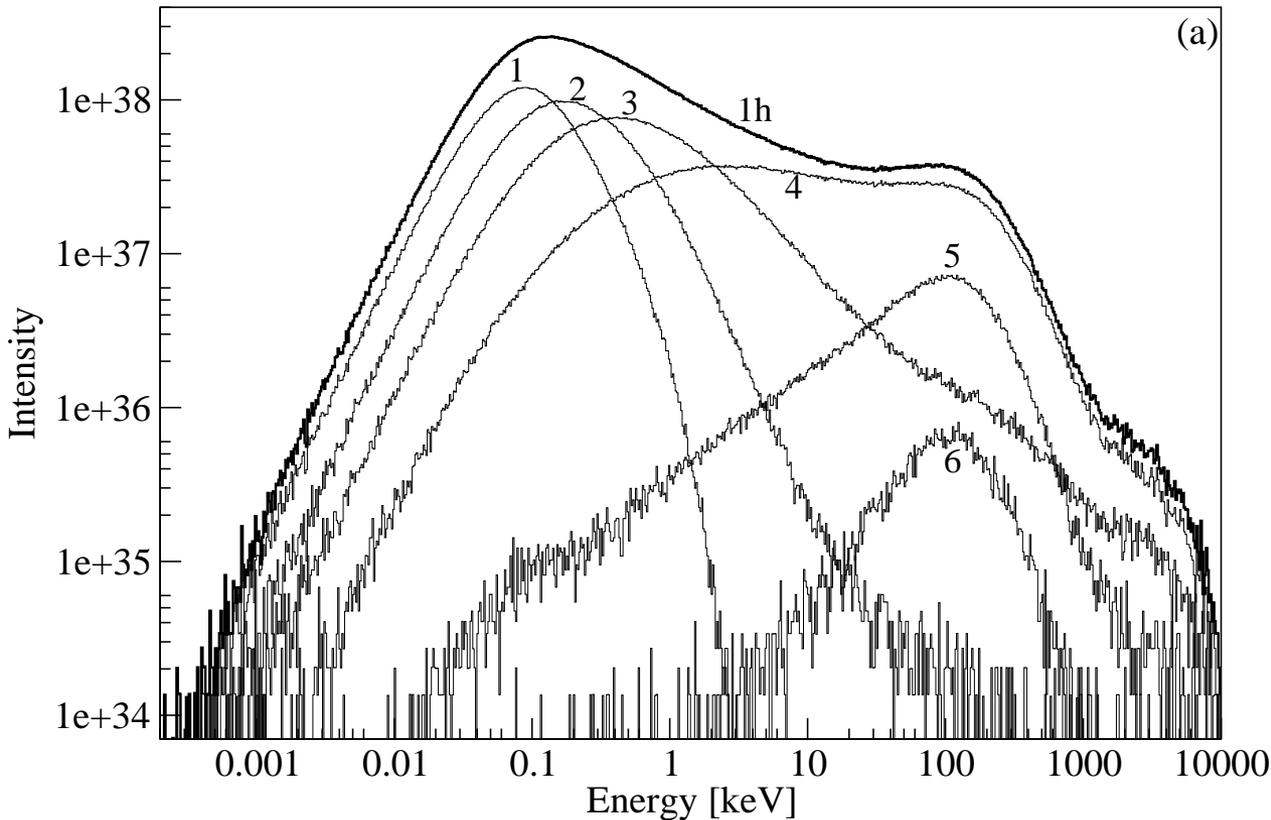
As we increase the disk rate  
keeping halo rate fixed, the  
spectrum becomes softer



As we increase the halo rate  
keeping disk rate fixed, the  
spectrum becomes harder



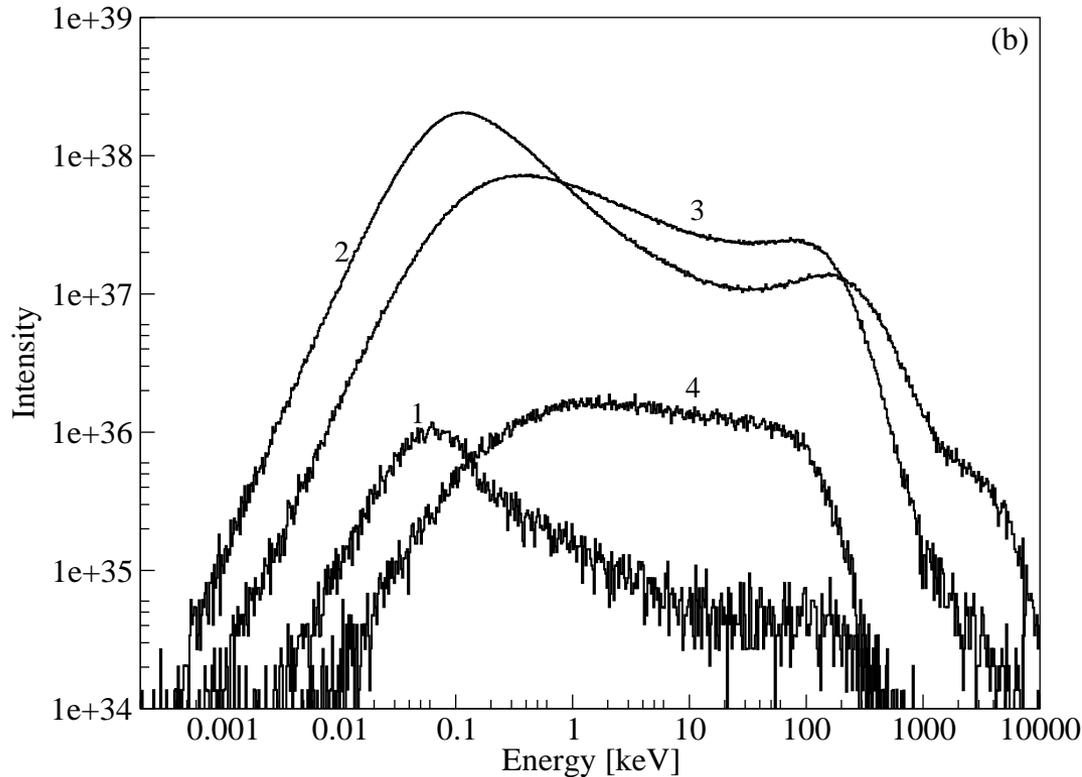
# Components of the net spectrum: spectrums of the photons, suffering different number of scatterings within the cloud



As the number of scattering increases, the photons get more energy from the electron cloud and become harder (spectrum 2, 3, 4). But, the photons which suffer more and more number of scatterings also spend more and more time inside the system. They start giving energy to electrons. Spectrum (5) and (6) are not inverse-Comptonised spectrum; it's the Compton scattered spectrum.

(1) 0, (2) 1-2, (3) 3-6, (4) 7-18, (5) 19-28, (6) 29 or higher, (h) Net output spectrum

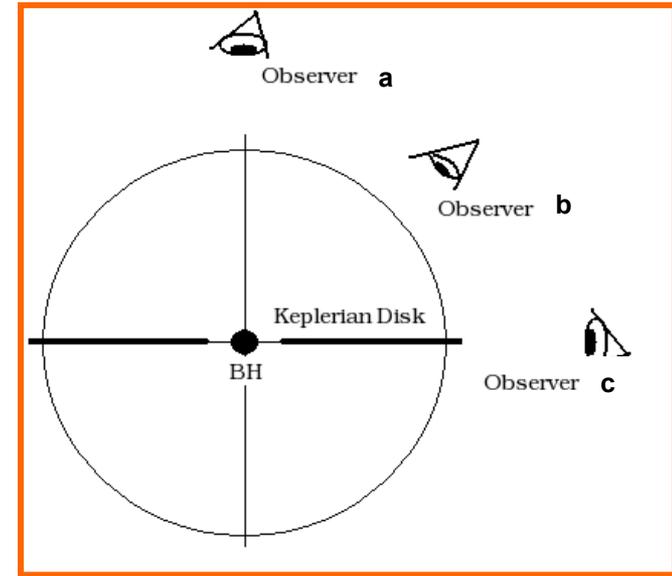
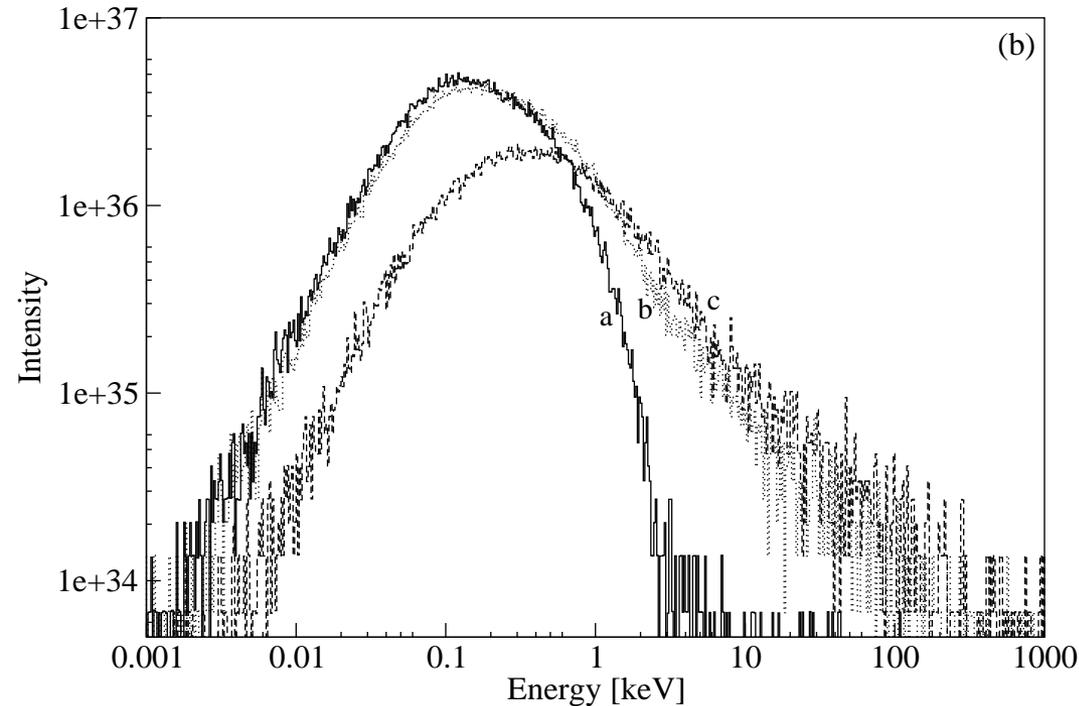
# Spectra of photons spending different times inside the electron cloud



As the photons spend more and more time inside the cloud it gets inverse-Comptonised to become harder (1, 2 & 3). But, after a certain time the photons start giving energy to the electrons. Spectrum 4 shows the photons have actually given Comptonised spectrum.

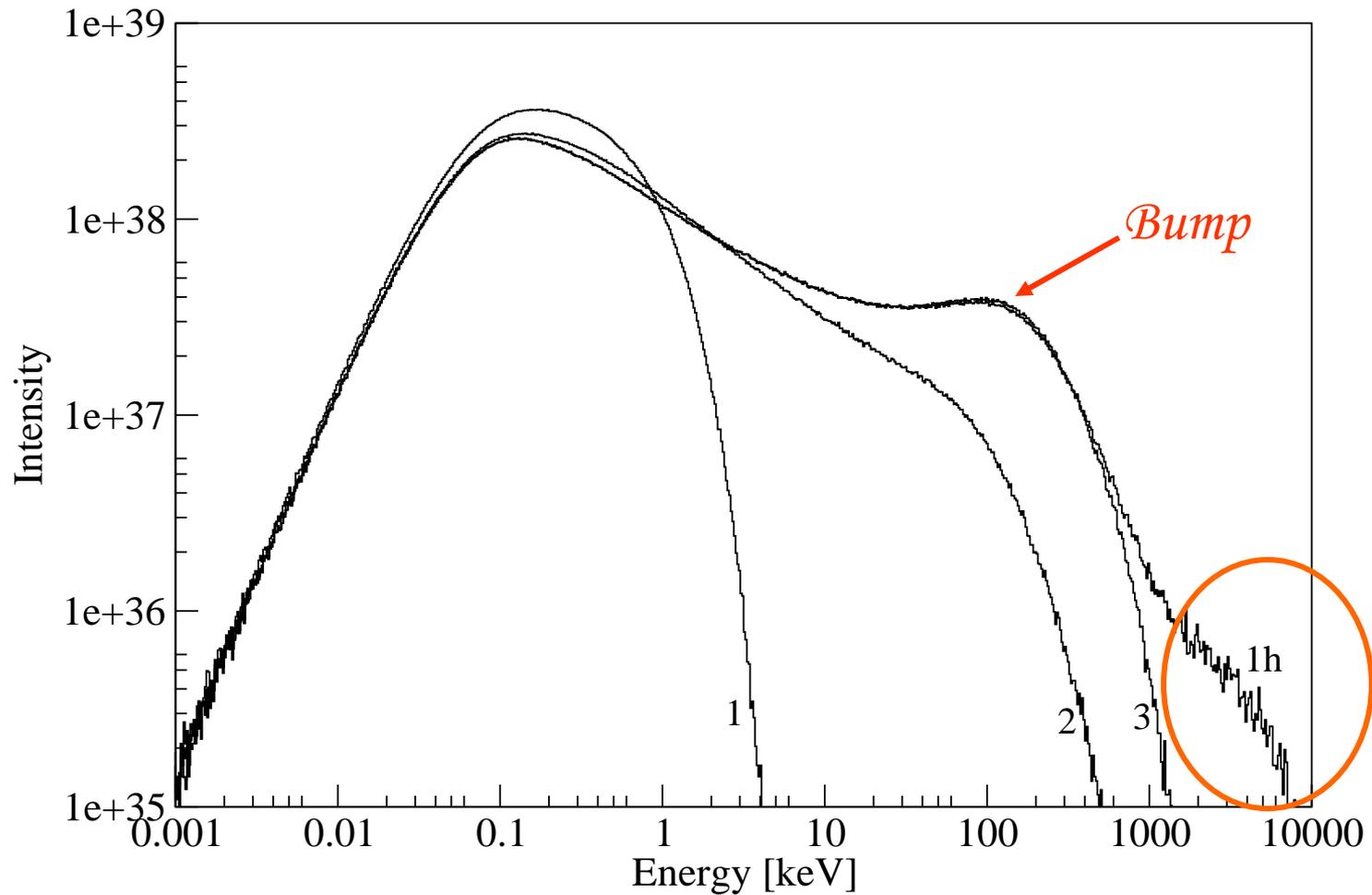
(1) 0.01 – 1 ms, (2) 1-40 ms, (3) 40-100 ms and (4) 100 ms or higher

# Directional properties of the spectrum



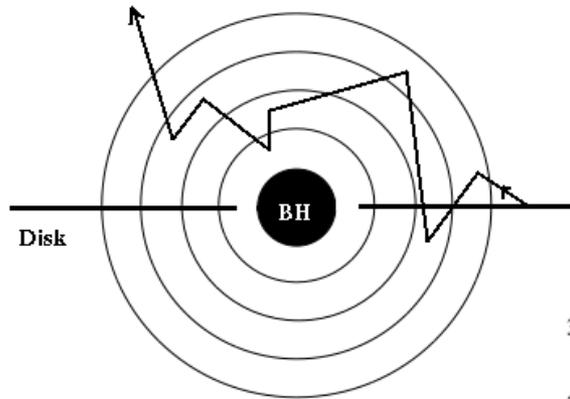
Variation of the output spectrum as viewed from different angles. Observing angles (in degree) with respect to the disk axis are 2 (plot 1), 45 (plot 2) & 90 (plot 3). The spectrum is getting harder as the disk axis is tilted away from the line of sight.

# Bulk Motion Comptonization

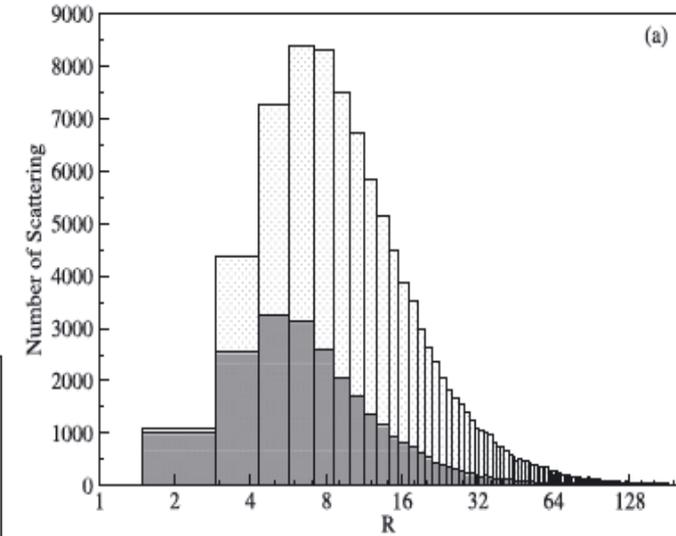


- (1) Injected spectrum (2) Static cloud (3) Cloud inside  $3r_g$  is static  
(1h) NET spectrum

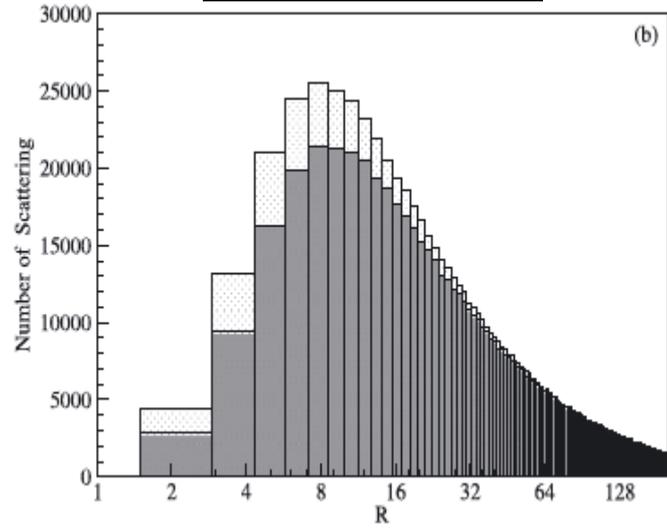
## Reason of occurrence of the Bump for higher halo rate



50 < E < 150 keV



All Photons



Number of scatterings inside different spherical shells before leaving the cloud.  
**Dark shade: static cloud, Light shade: spherically inflowing matter**

Bulk motion of the infalling electrons pushes the photons towards the hotter and denser inner region of the cloud to suffer more and more scatterings.

$$T_e (r=8r_g) \sim 100 \text{ keV}$$

# Flow with angular momentum

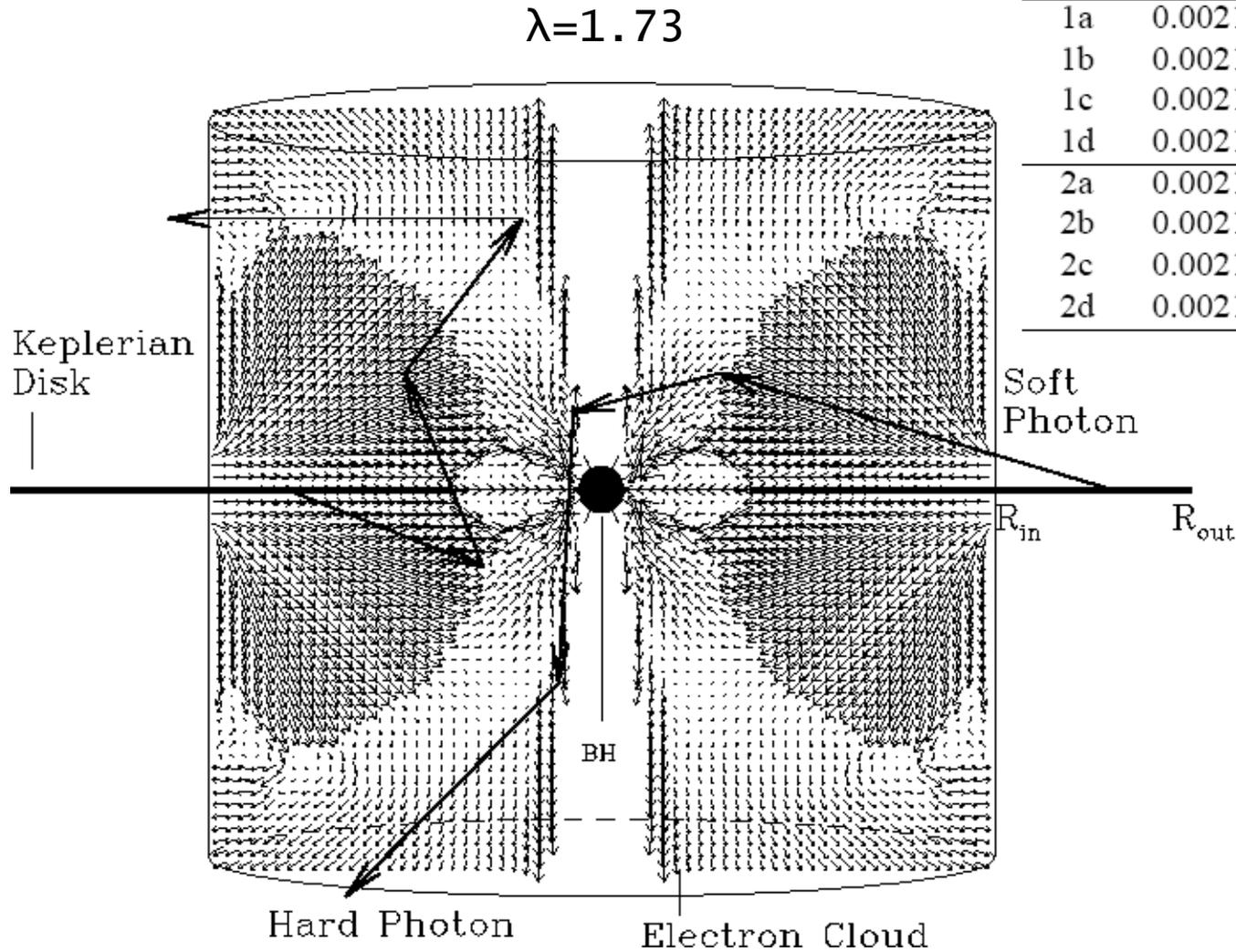


Table 1: Parameters used for the simulations.

Case	$\epsilon, \lambda$	$\dot{m}_h$	$\dot{m}_d$
1a	0.0021, 1.76	1.0	No Disk
1b	0.0021, 1.76	1.0	0.5
1c	0.0021, 1.76	1.0	1.0
1d	0.0021, 1.76	1.0	2.0
2a	0.0021, 1.73	1.0	No Disk
2b	0.0021, 1.73	1.0	0.5
2c	0.0021, 1.73	1.0	1.0
2d	0.0021, 1.73	1.0	2.0

$$M_{bh} = 10 M_{\odot}$$

$$R_{in} = 100 r_g$$

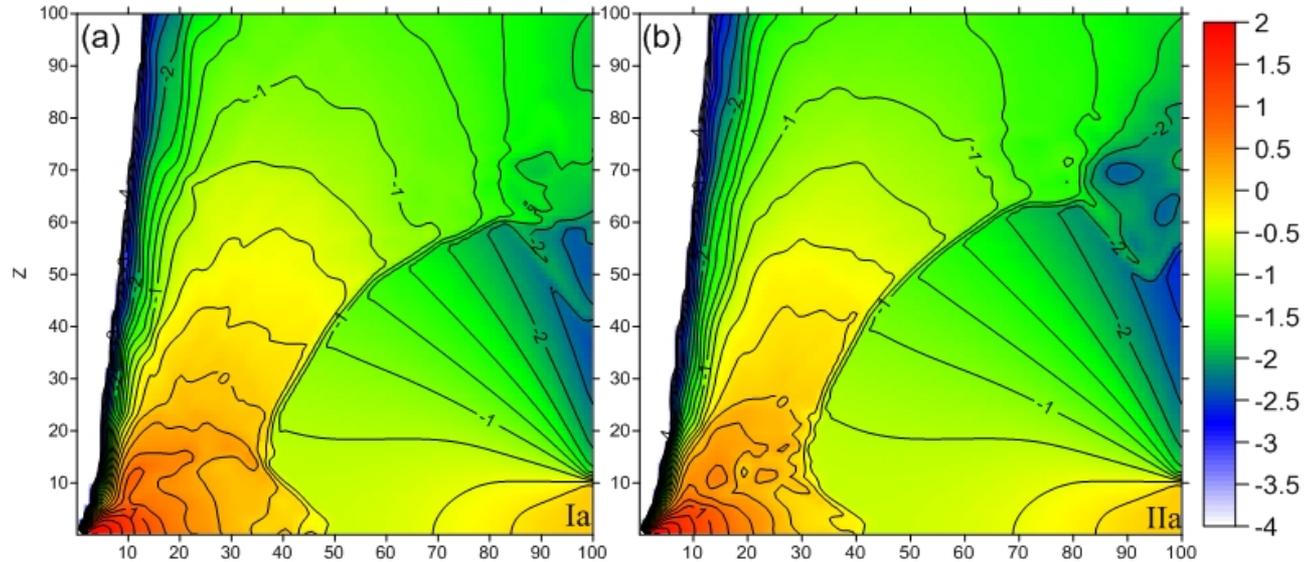
$$R_{out} = 200 r_g$$

# Initial Density & Temperature Distribution

$\lambda=1.76$

$\lambda=1.73$

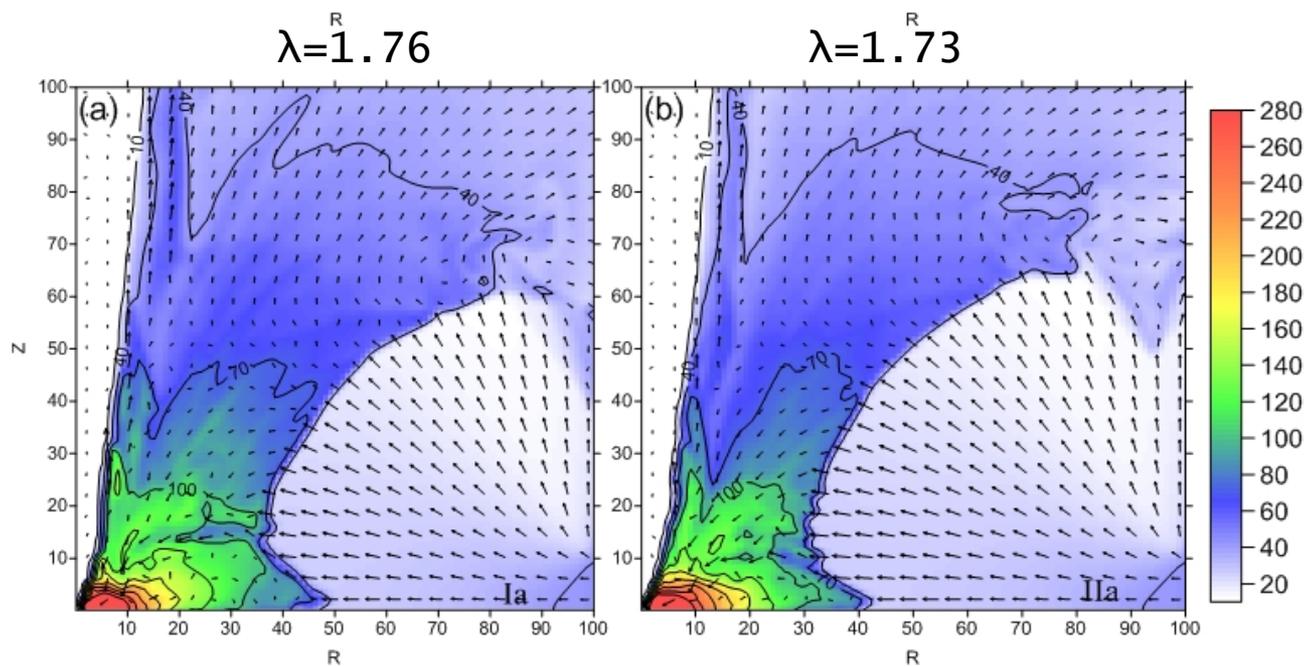
Density



$\lambda=1.76$

$\lambda=1.73$

Temperature

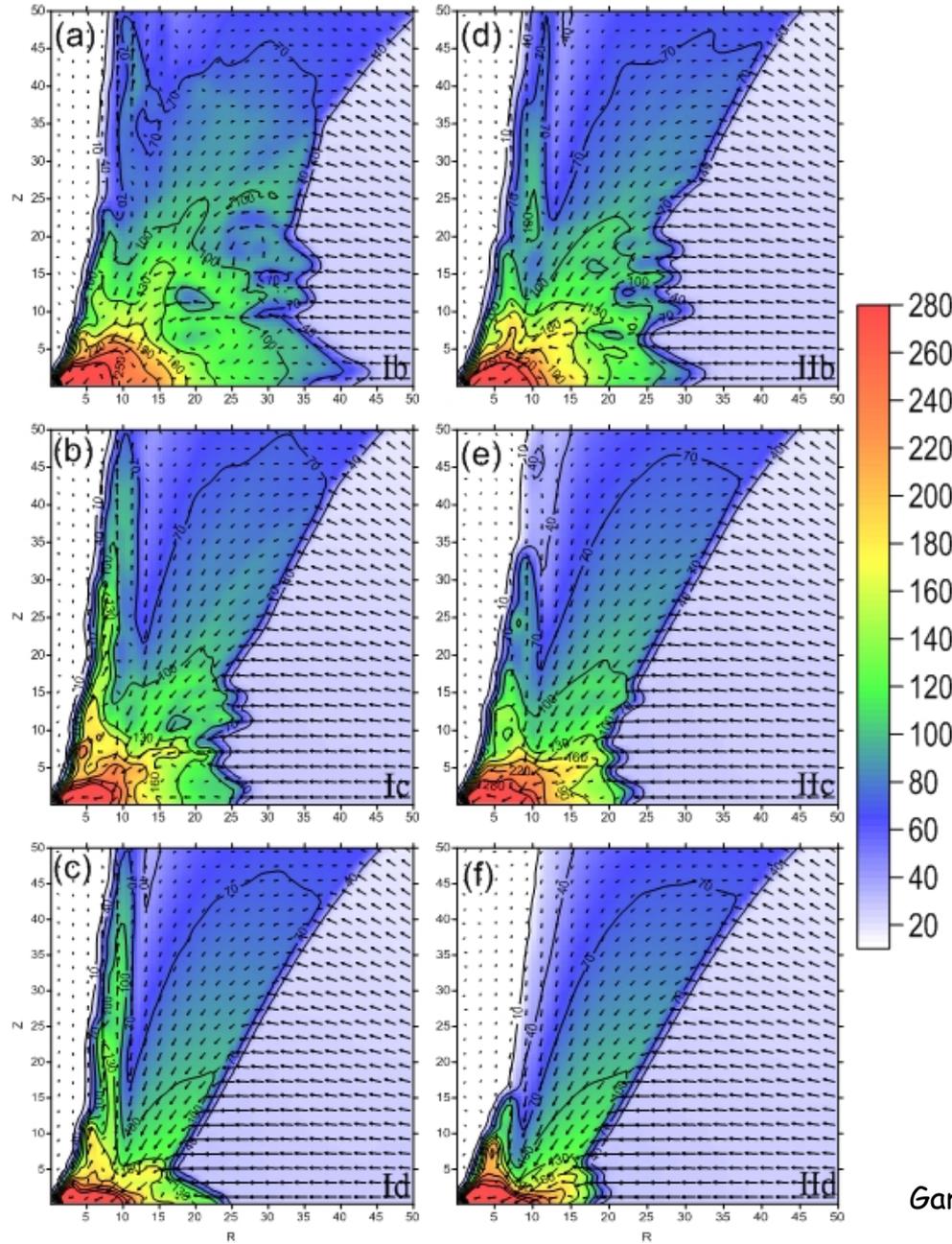


# Final Temperature (keV) of the Cloud: Compton Cooling

$\lambda=1.76$

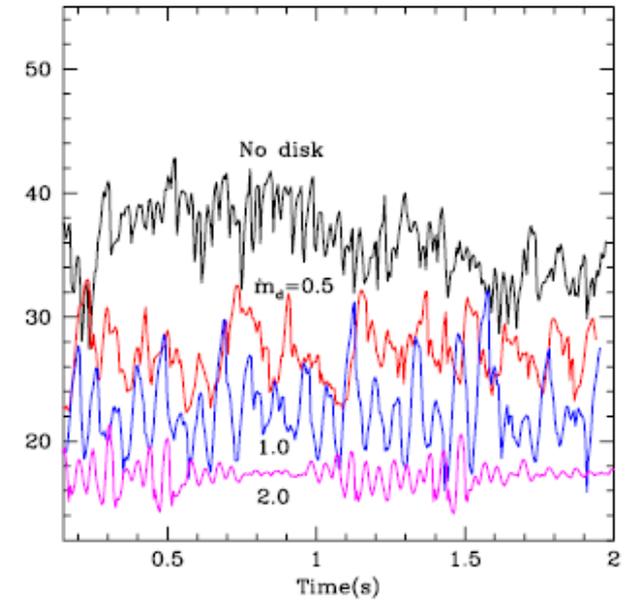
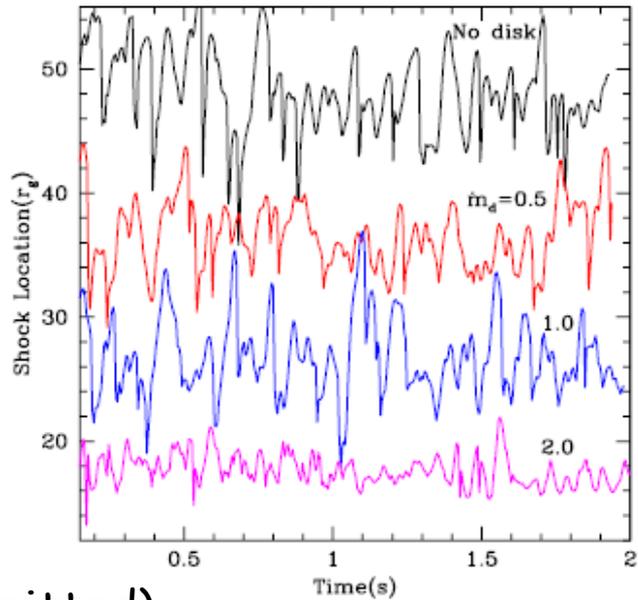
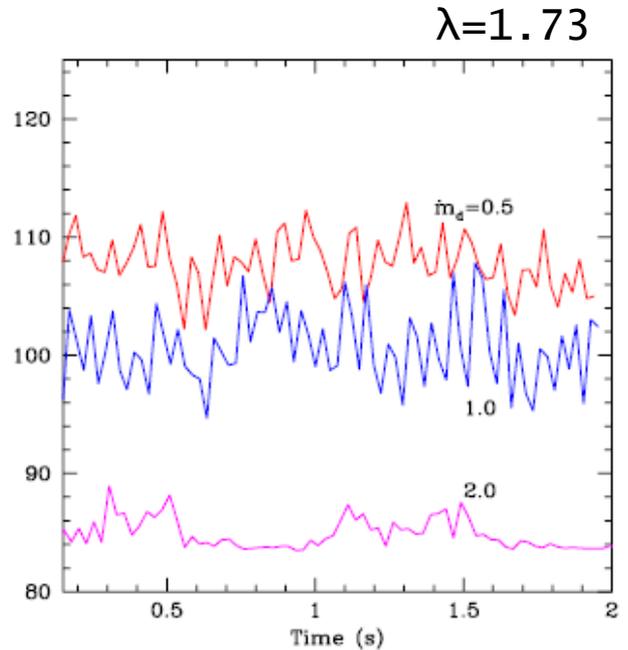
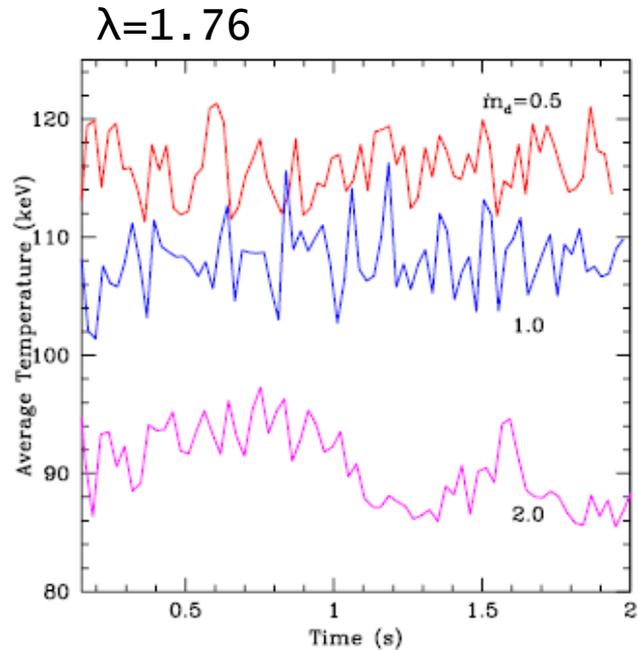
$\lambda=1.73$

As the disk accretion rate increases, the post-shock region of the electron cloud becomes cooler, due to the increase in the number of injected soft photons and the shock moves closer towards the black hole

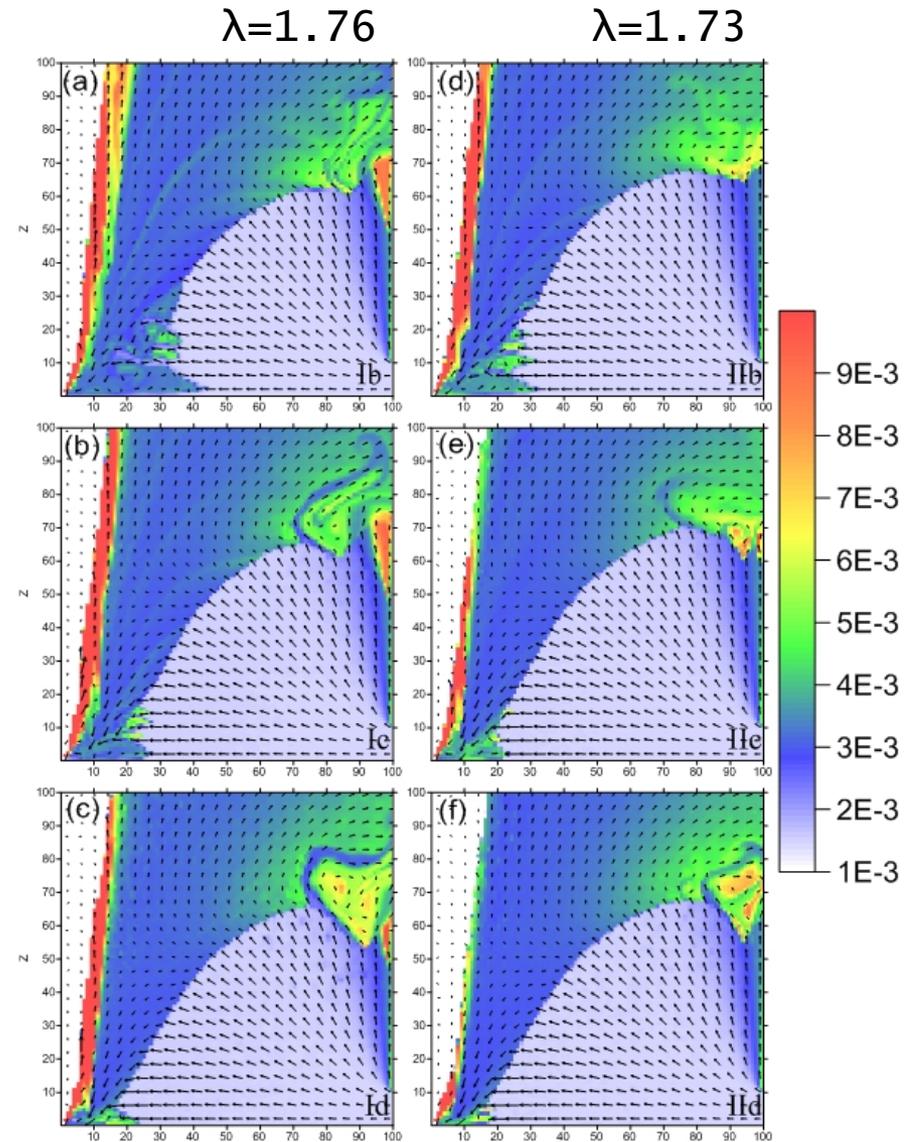
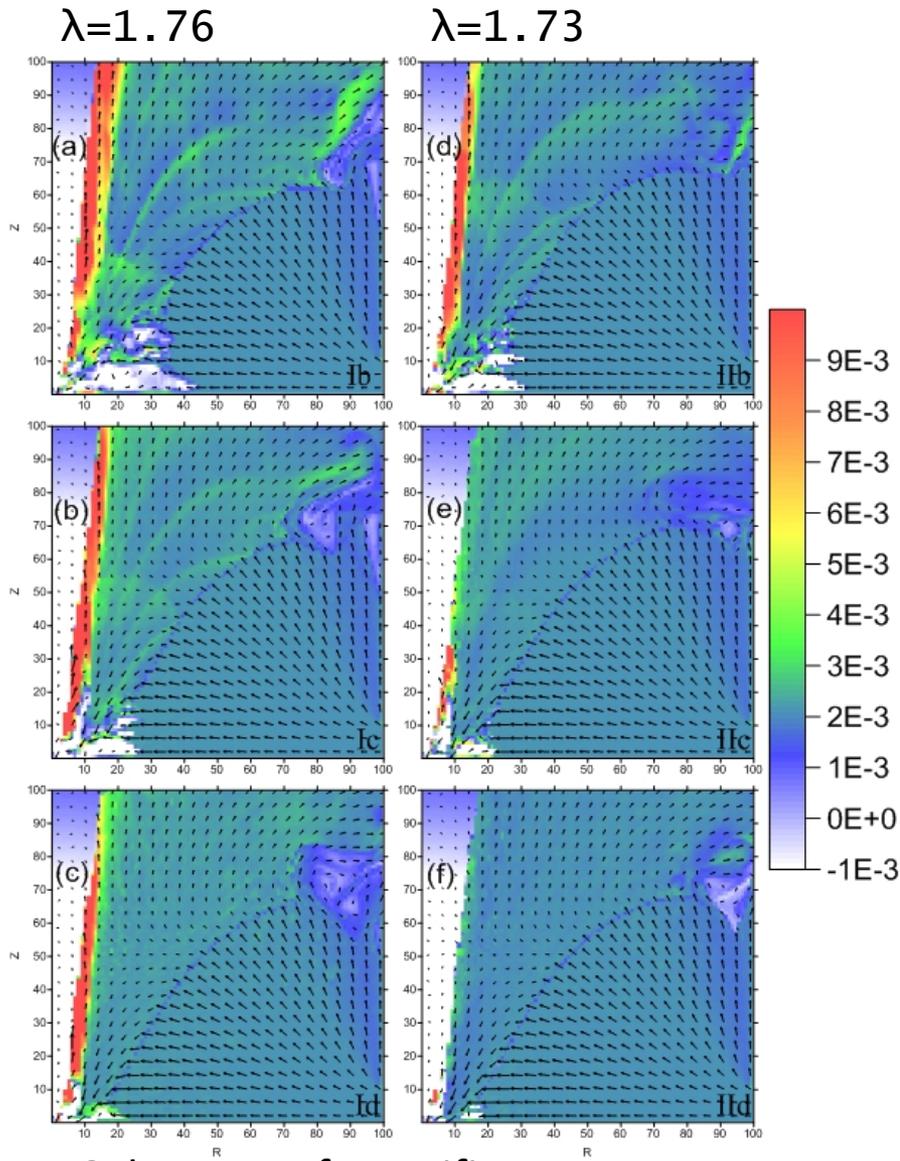


# Effects of Compton Cooling

As the post-shock region of the electron cloud becomes cooler, the shock moves closer towards the black hole



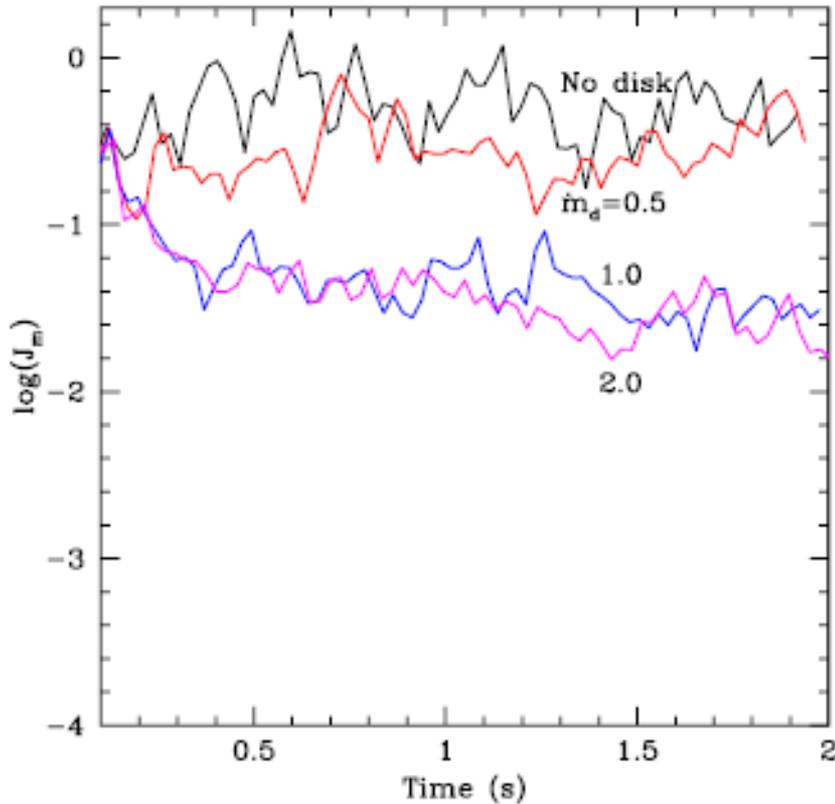
# Quenching of Jet



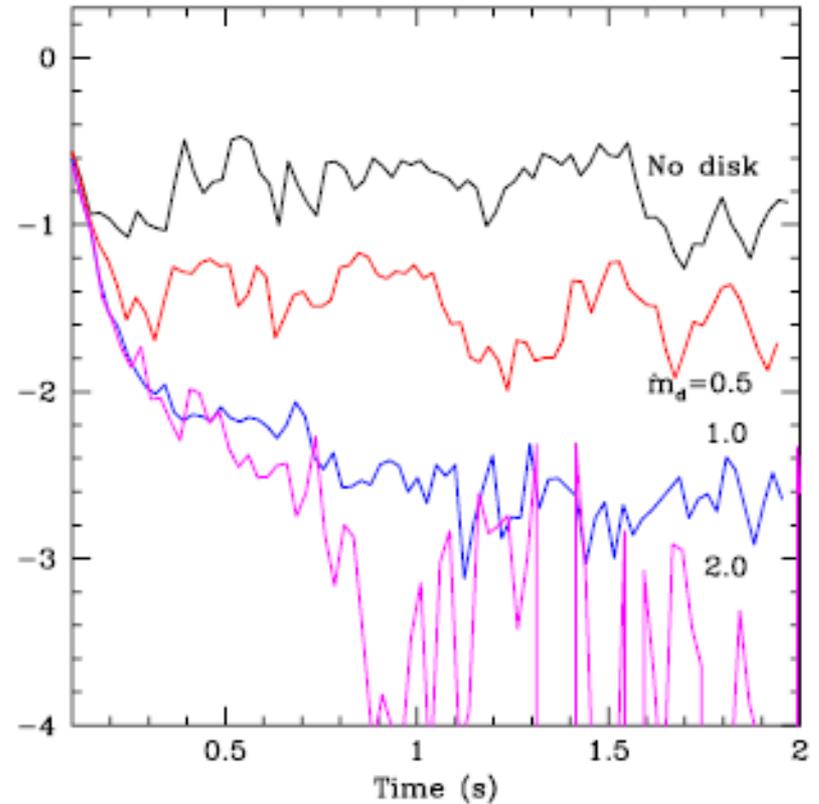
High energy (and entropy, dark red region) flow decrease as the disk rate increases i.e the hollow jet is quenched

# Outflow rate decreases as the disk rate is increased

$\lambda=1.76$

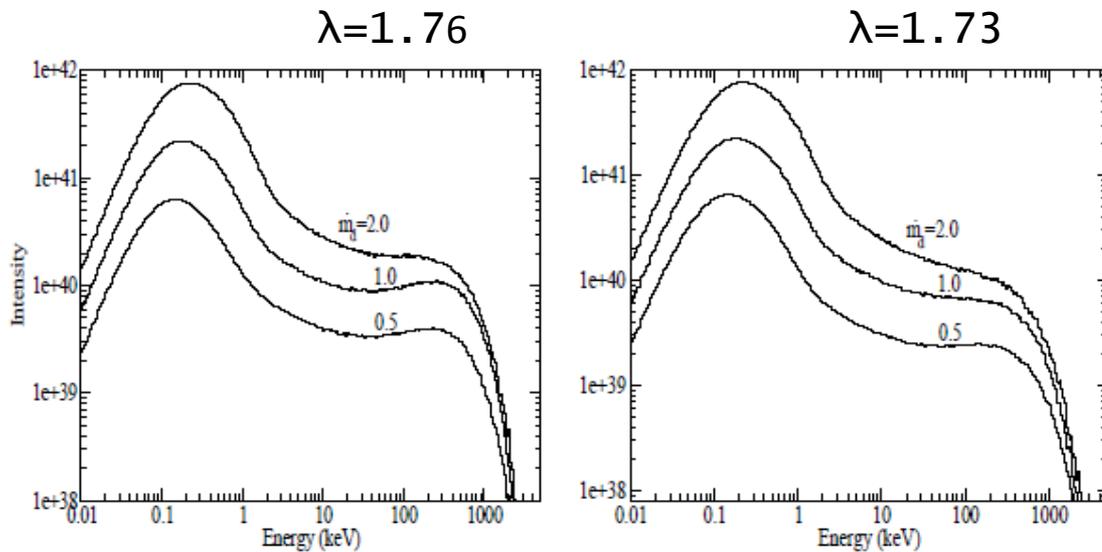


$\lambda=1.73$



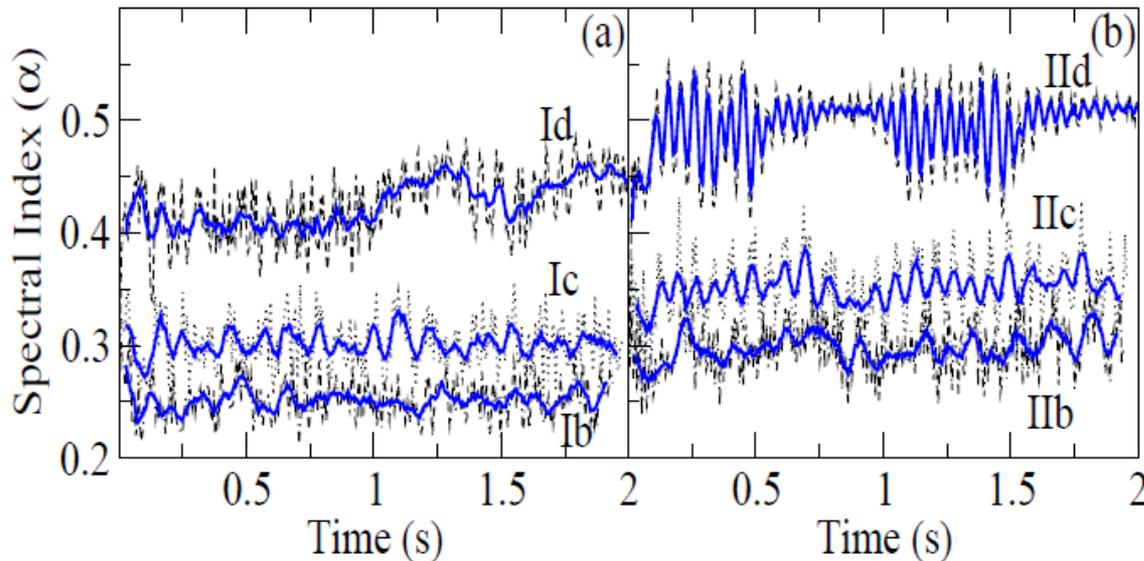
Variations of  $J_{in}$  ( $= \frac{\dot{M}_{jet}}{M_{in}} \times 100 \%$ ) with time for different  $m_d$

# Effects on Spectral Properties



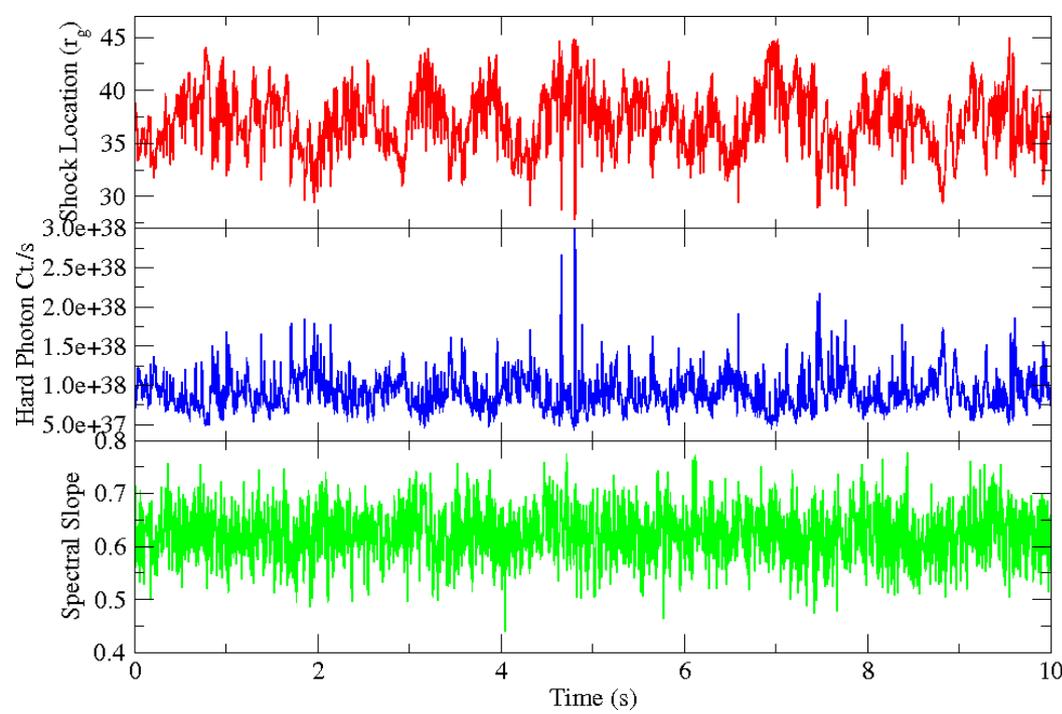
Spectrum softens  
as the disk rate  
is increased

The final output spectra for different  $\dot{m}_d$



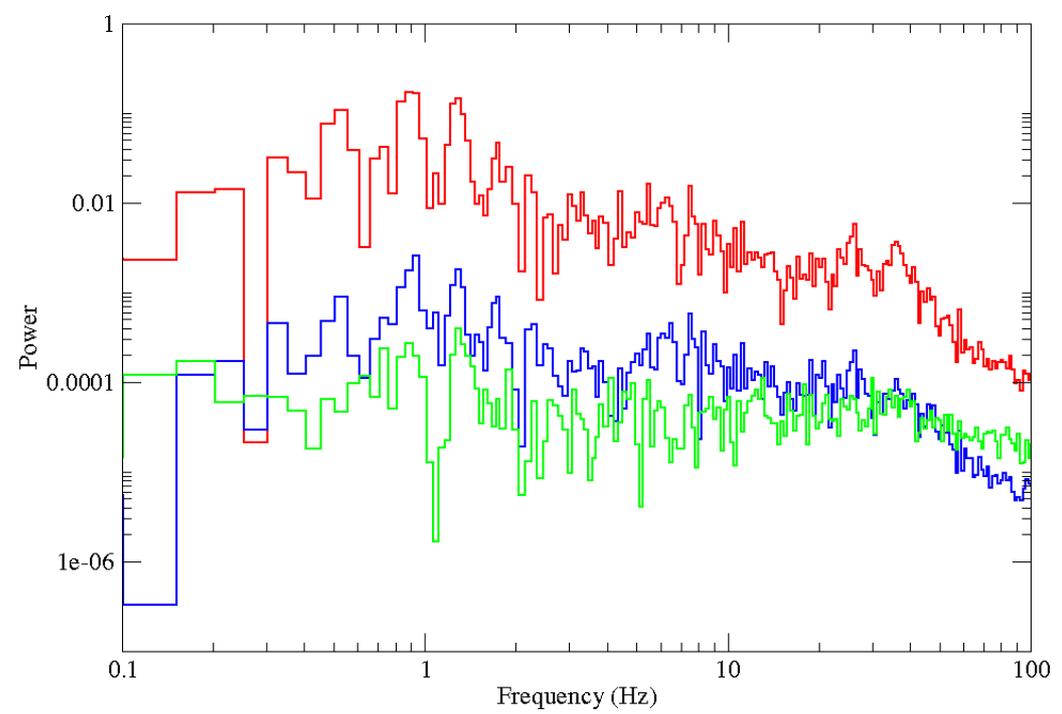
Garain et al. 2012 (submitted)

Time variation of the spectral slope ( $\alpha$ ,  $I(E) \propto E^{-\alpha}$ ) for different  $\dot{m}_d$



Disk rate= 0.1, Halo rate= 1  
 Hard photons=> 3 to 100 keV  
 Spectral slope is calculated  
 between 2 to 10 keV energy

The variation in shock location, Hard photon count and the spectral slope shows QPOs of similar frequencies



Ghosh et al. 2012 (in prep.)

# Summary & Conclusion

- Development of a time dependent coupled hydrodynamic - radiative transfer code
- Spherically symmetric Bondi accretion flow loses its spherical symmetry in presence of Compton cooling
- Directional and temporal properties of the output photons are seen
- For a high halo rate, even in the hard states, the bulk motion Comptonization leaves its mark as a power-law at high energies
- In presence of angular momentum, the accretion flow creates a boundary layer
- Increase in disk rate results in the increase of soft photons and hence, post shock region cools down more and more with increased disk rate
- As a result of Compton cooling, the thermal pressure goes down and shock forms closer to the black hole
- Outflow rate reduces and spectrum becomes softer as a result of cooling
- This shows a direct correlation between the spectral states and the outflow rates of an accreting black hole

Thank You!!!

velocity variations etc.) and the thermal properties of the sub-Keplerian matter. We assume the absorbing boundary condition at  $r = 1.5$ .

### 3.1. Details of the hydrodynamic simulation code

To model the initial injection of matter, we consider an axisymmetric flow of gas in the pseudo-Newtonian gravitational field of a black hole of mass  $M_{bh}$  located at the centre in the cylindrical coordinates  $[R, \theta, z]$ . We assume that at infinity, the gas pressure is negligible and the energy per unit mass vanishes. We also assume that the gravitational field of the black hole can be described by Paczyński & Wiita (1980),

$$\phi(r) = -\frac{GM_{bh}}{(r - r_g)},$$

where,  $r = \sqrt{R^2 + z^2}$ . We also assume a polytropic equation of state for the accreting (or, outflowing) matter,  $P = K\rho^\gamma$ , where,  $P$  and  $\rho$  are the isotropic pressure and the matter density respectively,  $\gamma$  is the adiabatic index (assumed to be constant throughout the flow, and is related to the polytropic index  $n$  by  $\gamma = 1 + 1/n$ ) and  $K$  is related to the specific entropy of the flow  $s$ . The details of the code is described in Ryu, Molteni & Chakrabarti (1997) and in Giri et al. (2010).

Our computational box occupies one quadrant of the  $R$ - $z$  plane with  $0 \leq R \leq 100$  and  $0 \leq z \leq 100$ . The incoming gas enters the box through the outer boundary, located at  $R_{in} = 100$ . We have chosen the density of the incoming gas  $\rho_{in} = 1$  for convenience since, in the absence of self-gravity and cooling, the density is scaled out, rendering the simulation results valid for any accretion rate. As we are considering only energy flows while keeping the boundary of the numerical grid at a finite distance, we need the sound speed  $a$  (i.e., temperature) of the flow and the incoming velocity at the boundary points. In order to mimic the horizon of the black hole at the Schwarzschild radius, we place an absorbing inner boundary at  $r = 1.5r_g$ , inside which all material is completely absorbed into the black hole. For the background matter (required to avoid division by zero) we used a stationary gas with density  $\rho_{bg} = 10^{-6}$  and sound speed (or temperature) the same as that of the incoming gas. Hence the incoming matter has a pressure  $10^6$  times larger than that of the background matter. All the calculations were performed with  $512 \times 512$  cells, so each grid has a size of 0.19 in units of the Schwarzschild radius.

### 3.2. Details of the radiative transfer code

To begin a Monte-Carlo simulation, we generate photons from the Keplerian disk with randomized locations as mentioned in the earlier section. The energy of the soft photons at radiation temperature  $T(r)$  is calculated using the Planck's distribution formula, where the number density of the photons ( $n_\gamma(E)$ ) having an energy  $E$  is expressed by,

$$n_\gamma(E) = \frac{1}{2\zeta(3)} b^3 E^2 (e^{bE} - 1)^{-1}, \quad (5)$$

where,  $b = 1/kT(r)$  and  $\zeta(3) = \sum_{l=1}^{\infty} l^{-3} = 1.202$ , the Riemann zeta function. Using another set of random numbers we ob-

tained the direction of the injected photon and with yet another random number we obtained a target optical depth  $\tau_c$  at which the scattering takes place. The photon was followed within the electron cloud till the optical depth ( $\tau$ ) reached  $\tau_c$ . The increase in optical depth ( $d\tau$ ) during its traveling of a path of length  $dl$  inside the electron cloud is given by:  $d\tau = \rho_n \sigma dl$ , where  $\rho_n$  is the electron number density.

The total scattering cross section  $\sigma$  is given by Klein-Nishina formula:

$$\sigma = \frac{2\pi r_e^2}{x} \left[ \left(1 - \frac{4}{x} - \frac{8}{x^2}\right) \ln(1+x) + \frac{1}{2} + \frac{8}{x} - \frac{1}{2(1+x)^2} \right], \quad (6)$$

where,  $x$  is given by,

$$x = \frac{2E}{mc^2} \gamma \left(1 - \mu \frac{v}{c}\right), \quad (7)$$

$r_e = e^2/mc^2$  is the classical electron radius and  $m$  is the mass of the electron.

We have assumed here that a photon of energy  $E$  and momentum  $\frac{E}{c}\hat{\Omega}$  is scattered by an electron of energy  $\gamma mc^2$  and momentum  $\vec{p} = \gamma m\vec{v}$ , with  $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$  and  $\mu = \hat{\Omega} \cdot \hat{v}$ . At this point, a scattering is allowed to take place. The photon selects an electron and the energy exchange is computed using the Compton or inverse Compton scattering formula. The electrons are assumed to obey relativistic Maxwell distribution inside the Compton cloud. The number  $dN(p)$  of Maxwellian electrons having momentum between  $p$  to  $p + dp$  is expressed by,

$$dN(p) \propto \exp[-(p^2 c^2 + m^2 c^4)^{1/2} / kT_e] dp. \quad (8)$$

#### 3.2.1. Calculation of energy reduction using Monte Carlo code:

We divide the Keplerian disk in different annuli of width  $D(r) = 0.5$ . Each annulus is characterized by its central temperature  $T(r)$ . The total number of photons emitted from the disk surface of each annulus can be calculated using Eq. 3. This total number comes out to be  $\sim 10^{41-42}$  per second for  $\dot{m}_a = 1.0$ . In reality, one cannot inject this much number of photons in Monte Carlo simulation because of the limitation of computation time. So we replace this large number of photons by a low number of bundles, say,  $N_{comp}(r) \sim 10^7$  and calculate a weightage factor

$$f_W = \frac{dN(r)}{N_{comp}(r)}.$$

Clearly, from each annulus, the number of photons in a bundle will vary. This is computed exactly and used to compute the change of energy due to Comptonization. When this injected photon is inverse-Comptonized (or, Comptonized) by an electron in a volume element of size  $dV$ , we assume that  $f_W$  number of photons has suffered similar scattering with the electrons inside the volume element  $dV$ . If the energy loss (gain) per electron in this scattering is  $\Delta E$ , we multiply this amount by  $f_W$  and distribute this loss (gain) among all the electrons inside that particular volume element. This is continued for all the  $10^7$  bundles of photons and the revised energy distribution is obtained.

### 3.2.2. Computation of the temperature distribution after cooling

Since the hydrogen plasma considered here is ultra-relativistic ( $\gamma = \frac{4}{3}$  throughout the hydrodynamic simulation), thermal energy per particle is  $3k_B T$  where  $k_B$  is Boltzmann constant,  $T$  is the temperature of the particle. The electrons are cooled by the inverse-Comptonization of the soft photons emitted from the Keplerian disk. The protons are cooled because of the Coulomb coupling with the electrons. Total number of electrons inside any box with the centre at location  $(ir, iz)$  is given by,

$$dN_e(ir, iz) = 4\pi r n_e(ir, iz) dr dz, \quad (9)$$

where,  $n_e(ir, iz)$  is the electron number density at  $(ir, iz)$  location, and  $dr$  and  $dz$  represent the grid size along  $r$  and  $z$  directions respectively. So the total thermal energy in any box is given by  $3k_B T(ir, iz) dN_e(ir, iz) = 12\pi r k_B T(ir, iz) n_e(ir, iz) dr dz$ , where  $T(ir, iz)$  is the temperature at  $(ir, iz)$  grid. We calculate the total energy loss (gain)  $\Delta E$  of electrons inside the box according to what is presented above and subtract that amount to get the new temperature of the electrons inside that box as

$$k_B T_{new}(ir, iz) = k_B T_{old}(ir, iz) - \frac{\Delta E}{3dN_e(ir, iz)}. \quad (10)$$

### 3.3. Coupling procedure

The hydrodynamic and the radiative transfer codes are coupled together following the same procedure as in Ghosh et al., 2011. Once a steady state is achieved in the non-radiative hydro-code, we compute the spectrum using the Monte Carlo code. This is the spectrum in the first approximation. To include cooling in the coupled code, we follow these steps: (i) we calculate the velocity, density and temperature profiles of the electron cloud from the output of the hydro-code. (ii) Using the Monte Carlo code we calculate the spectrum. (iii) Electrons are cooled (heated up) by the inverse-Compton (Compton) scattering. We calculate the amount of heat loss (gain) by the electrons and its new temperature and energy distributions and (iv) taking the new temperature and energy profiles as initial condition, we run the hydro-code for a period of time. Subsequently, we repeat the steps (i-iv). In this way we see how the spectrum is modified as the iterations proceed.

All the simulations are carried out assuming a stellar mass black hole ( $M_{bh} = 10M_\odot$ ). The procedures remain equally valid for massive/super-massive black holes. We carry out the simulations till several dynamical time-scales (more than 10) are passed. In reality, this corresponds to a few seconds in physical units.

