On the mass radiated by coalescing black hole binaries

E. Barausse, V. Morozova, L. Rezzolla

MPI for Gravitational Physics, Potsdam, Germany
University of Guelph, Ontario, Canada

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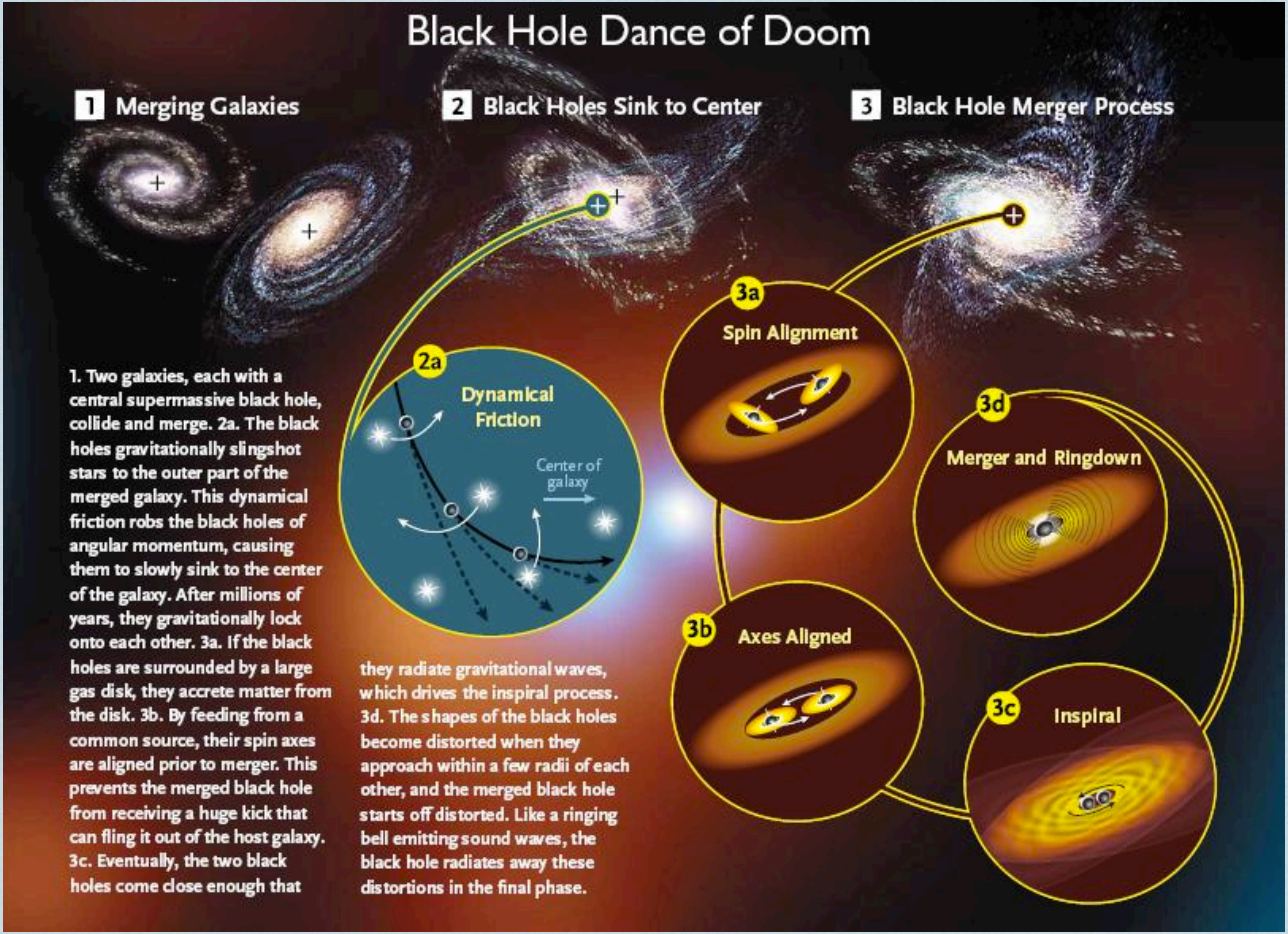
Plan of talk

- Introduction
- Derivation of the formula
- Verification of the formula
- Applications
- Conclusions

Introduction



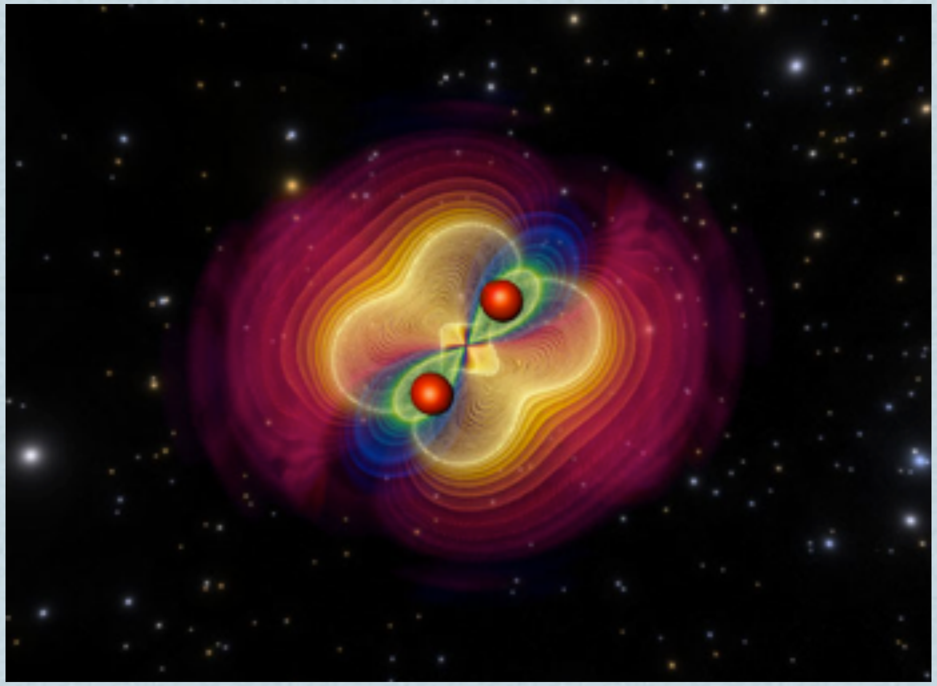
Introduction

- Black-hole mergers play a central role in gravitational-wave astrophysics, because they are expected to be among the main sources for existing and future detectors

Numerical simulations

40 years of intense research ended up with the first simulation of the final orbit, merger and ringdown of the black hole binary in 2005 by Pretorius

- Einstein equations are solved with high accuracy
- It is possible to get gravitational wave templates, electromagnetic counterparts, derive final parameters of the remnant
- Beautiful visualization of the data
- Expensive...



Another approaches?

Post-Newtonian models

Effective-one-body model

Parameters of the remnant black hole

- Final mass of the remnant
- Final spin of the remnant
- Recoil velocity of the remnant

Tools used to get the formulae for these parameters

- Hints from the Post-Newtonian theory
- Fitting the data of numerical simulations
- Investigation of the limiting cases
- Exploiting symmetries of the systems

Equal mass case

- The most accurately studied black hole binary systems to date are the equal-mass binaries with equal spins aligned/antialigned with the orbital angular momentum

- In the work of Reisswig et al. (2009) it was found that the energy emitted by these binaries during their inspiral, merger and ringdown can be well described by a polynomial fit

$$E_{rad}/M = p_0 + p_1(a_1 + a_2) + p_2(a_1 + a_2)^2$$

- A maximal value of the radiated energy evaluated from this formula is equal to 9.9% and corresponds to maximally rotating black holes with spins aligned with the orbital angular momentum of the binary

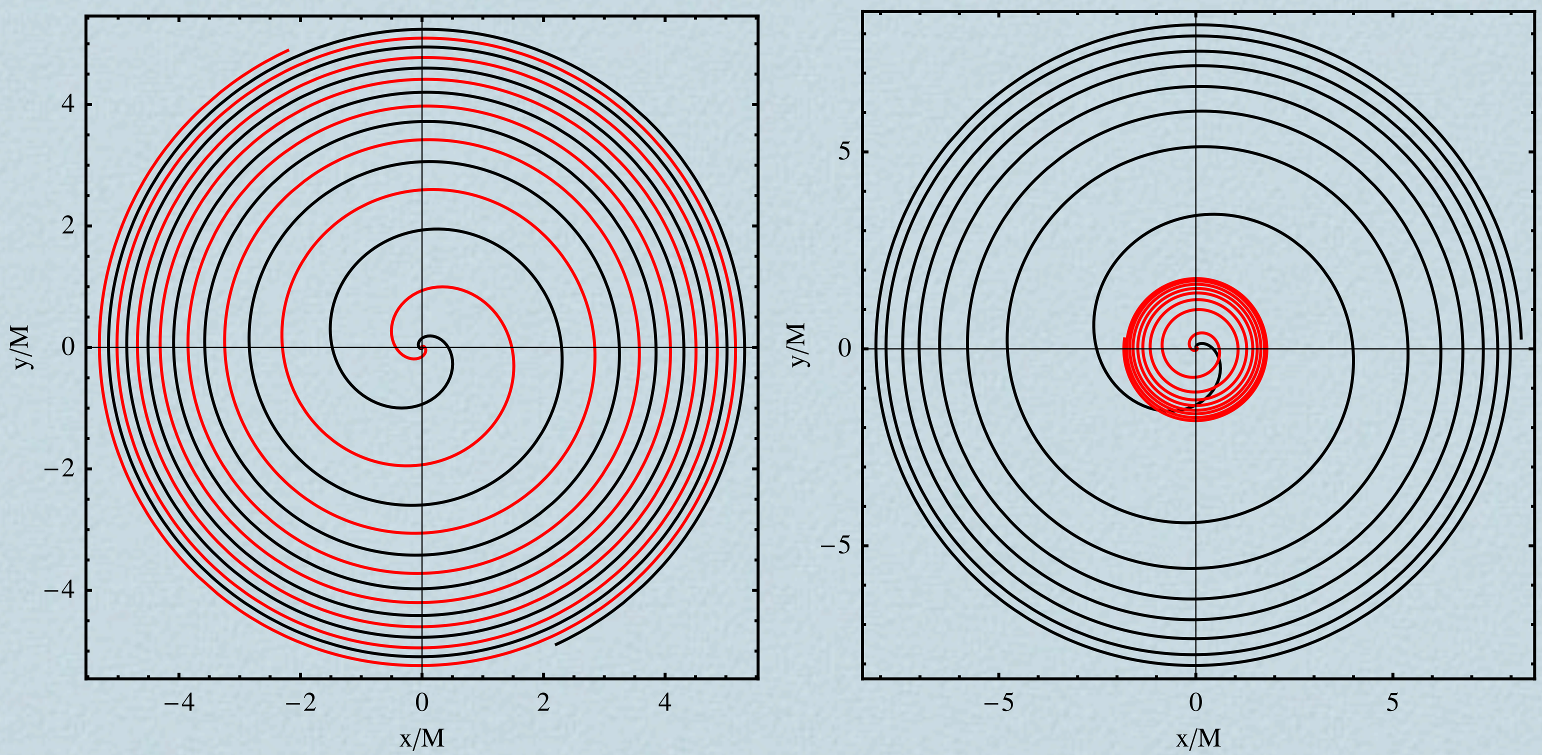
Derivation of the formula

Test-particle limit

- A test particle orbits around a BH through a sequence of circular orbits, slowly shrinking due to the emission of gravitational waves, until it reaches the innermost stable circular orbit (ISCO), where it starts plunging to the BH

- The energy emitted by the particle during the inspiral to the moment it merges with the central BH can be written as

$$E_{rad}/M = [1 - \tilde{E}_{ISCO}(a)]\eta + o(\eta) \qquad \tilde{E}_{ISCO}(a) = \sqrt{1 - \frac{2}{3\tilde{r}_{ISCO}(a)}}$$



$$\eta \equiv \frac{m_1 m_2}{M^2}$$

Hannam et al. (2010)

Orbital motion of non-spinning BHs with mass ratio 1 (left) and 4 (right)

New formula for the radiated energy

- We construct the generalized formula for the energy radiated during the inspiral, merger and ringdown of the black hole binary by
  - replacing $$a_1 + a_2 \to \tilde{a} = \frac{\hat{\mathbf{L}} \cdot (\mathbf{S}_1 + \mathbf{S}_2)}{M^2} = \frac{(|\mathbf{a}_1| \cos\beta + q^2 |\mathbf{a}_2| \cos\gamma)}{(1+q)^2}$$

    where $$\mathbf{a}_1 = \mathbf{S}_1/M_1^2 \ , \quad \mathbf{a}_2 = \mathbf{S}_2/M_2^2 \ , \quad q = M_2/M_1$$

    (based on the spin dependence of post-Newtonian binding energy)
  - requiring the formula to have correct limits in the equal-mass binary case and in the test-particle limit

$$\boxed{\begin{gathered} E_{rad}/M = [1 - \tilde{E}_{ISCO}(\tilde{a})]\eta + 4[4p_0 + 16p_1 \tilde{a}(\tilde{a}+1) + \tilde{E}_{ISCO}(\tilde{a}) - 1]\eta^2 \\ \tilde{E}_{ISCO}(\tilde{a}) = \sqrt{1 - \frac{2}{3\tilde{r}_{ISCO}(\tilde{a})}} \end{gathered}}$$

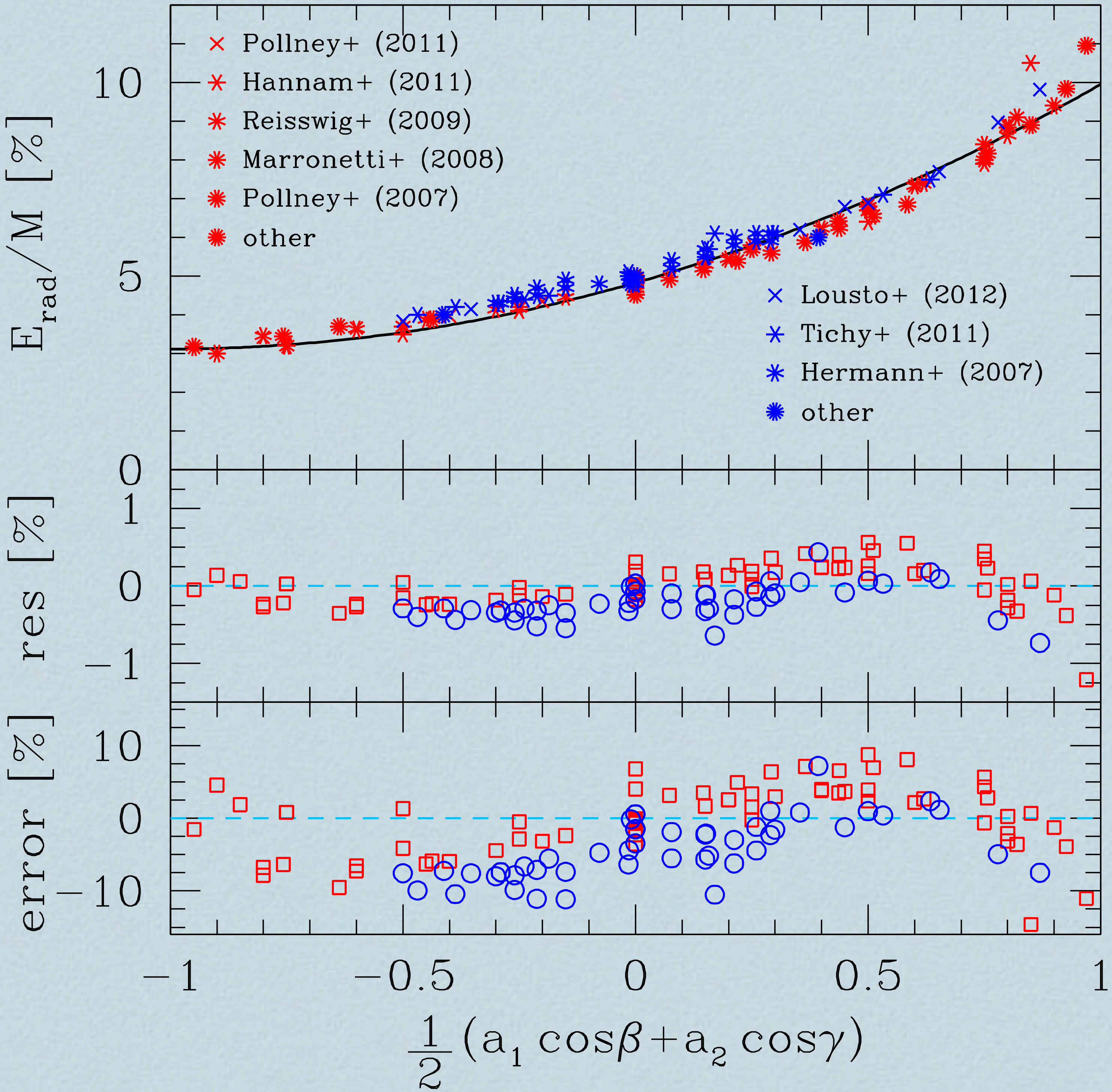
Numerical data

- We have collected the data of 186 numerical simulations (from 20 publications in the period 2006-2012) for the final mass of the remnant in terms of the initial mass of the system $M_{fin}/M = 1 - E_{rad}/M$ for the black hole binaries with generic initial spins and mass ratio

  - In cases where the energy radiated during the numerical simulations was provided, we reconstructed the ratio M_{fin}/M using $E_{rad} = E_{rad}^{NR} + |E_{3PN}(\Omega_0)|$ where $E_{3PN}(\Omega)$ is the 3PN binding energy as a function of orbital frequency Ω, and Ω_0 is the initial orbital frequency of the simulations

  - In cases where the simulation results were normalized in terms of the Arnowitt-Deser-Misner mass M_{ADM}, we approximated it as $M_{ADM} = M - |E_{3PN}(\Omega_0)|$

Fit of the equal-mass binaries



- We used the fitting formula

$$\frac{E_{rad}}{M} = p_0 + p_1 \tilde{a} + \frac{p_1}{4} \tilde{a}^2$$

enforcing the monotonicity of E_{rad}, i.e. placing the minimum at the point

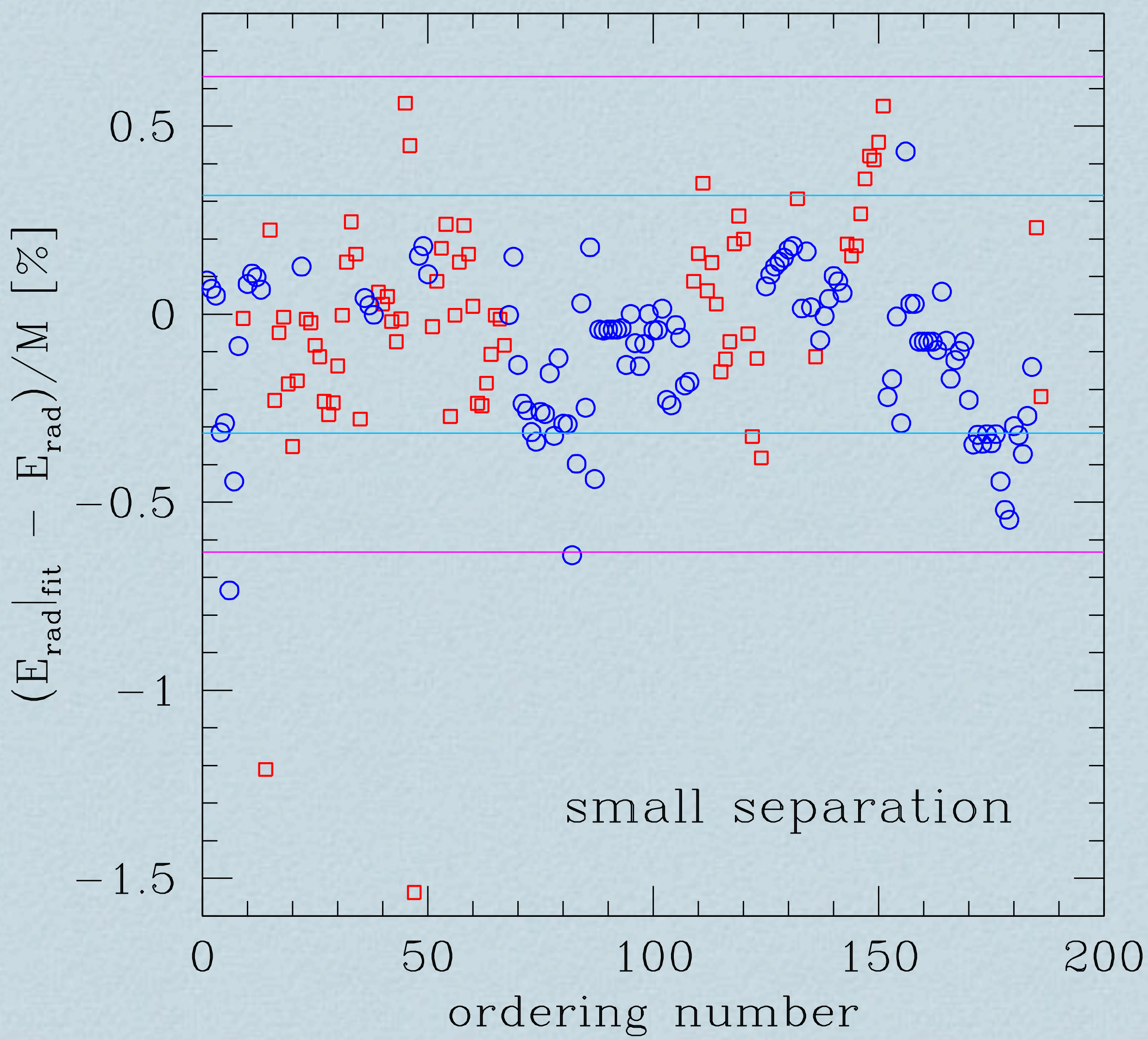
$$\tilde{a} = -2$$

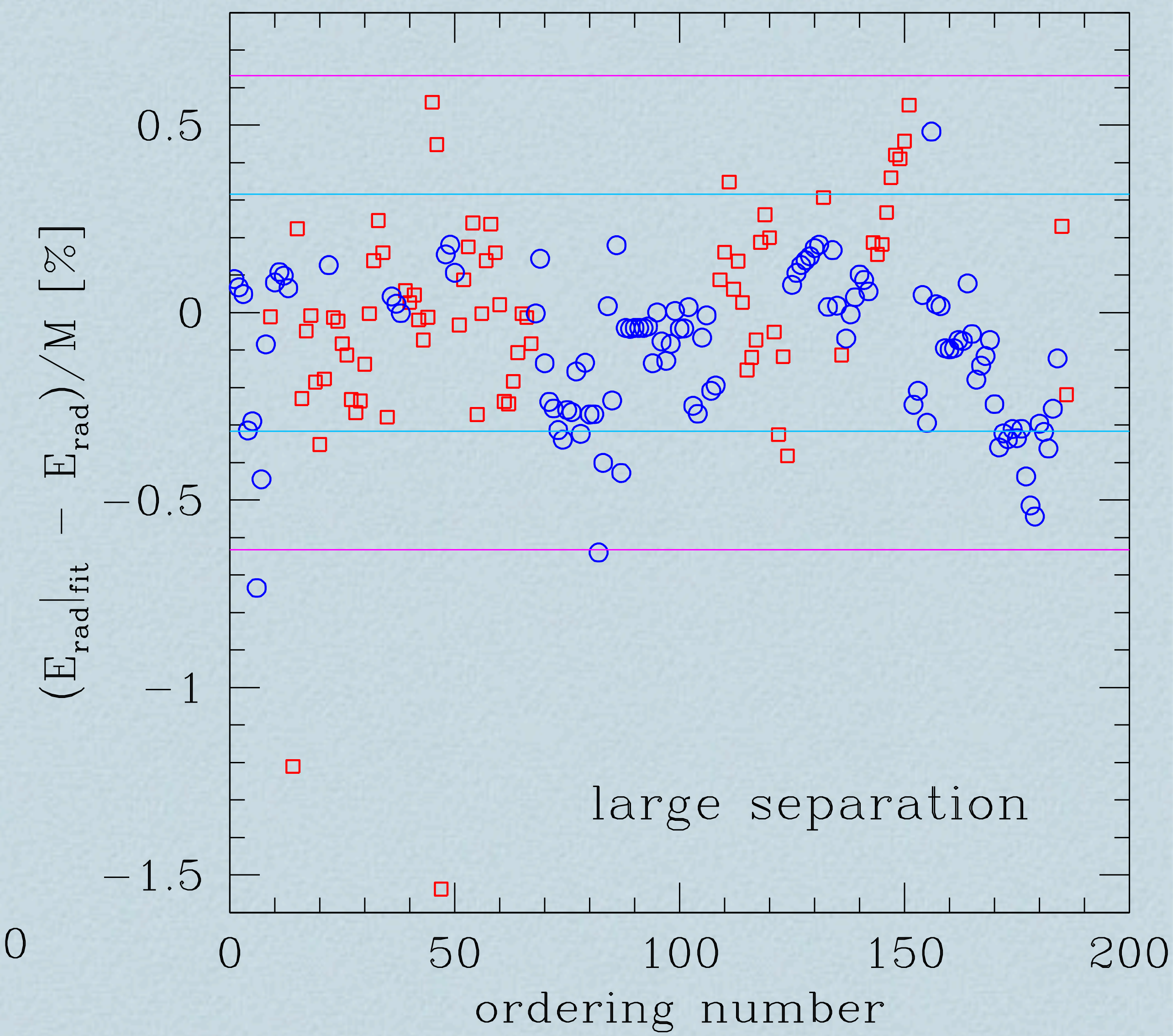
- The accuracy of the fit is comparable to the typical accuracy of the data themselves

Verification of the formula

Residuals of the data

- Small initial separation (initial configuration of the simulations)
- Large initial separation (after PN back integration)



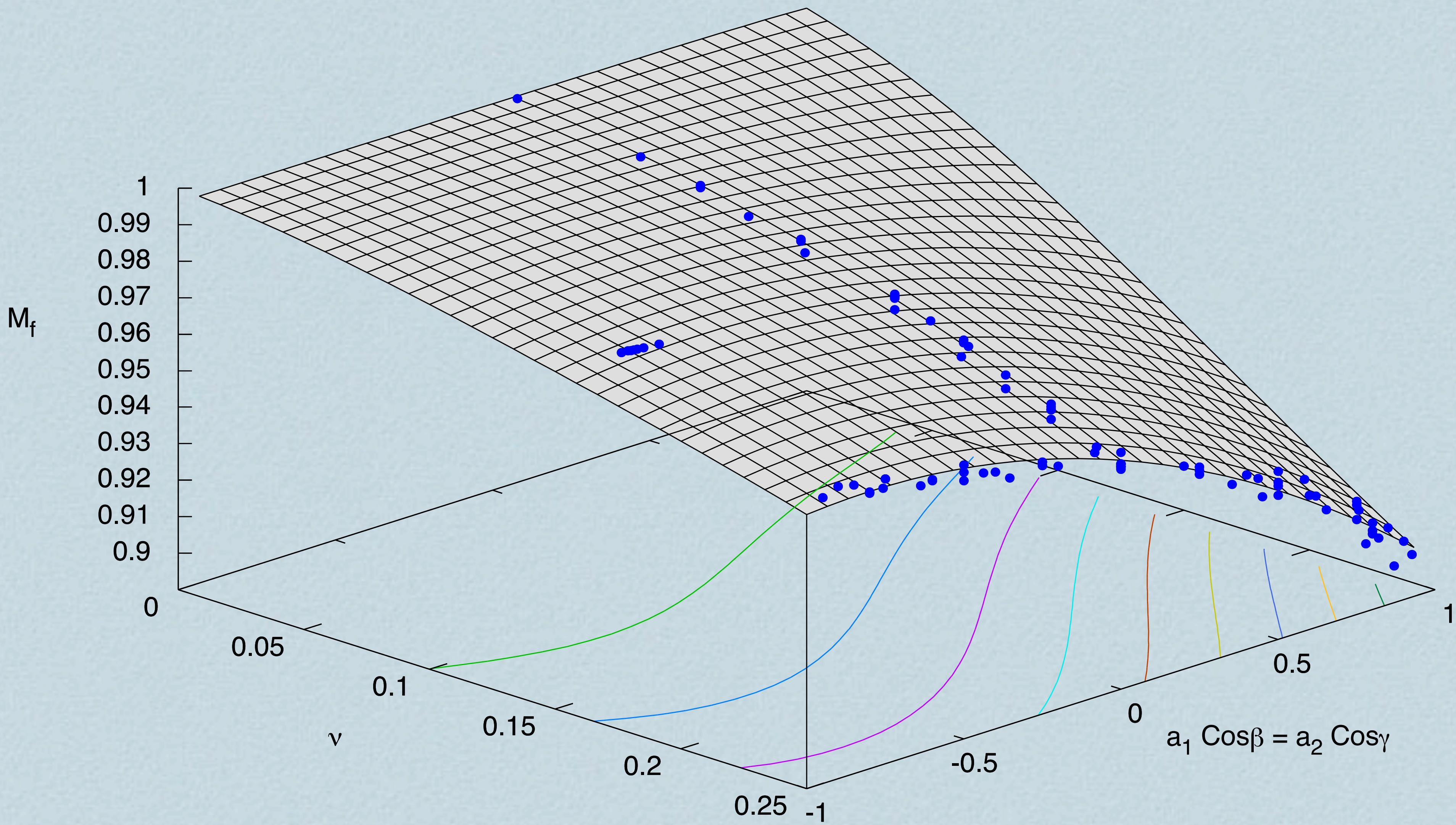


- First 50 points correspond to the simulations performed after 2010, while others correspond to the simulations performed in 2006-2009

Verification of the formula

Equal-spin binaries

- Numerical data together with the fitting formula for the binaries with equal z-components of the spins



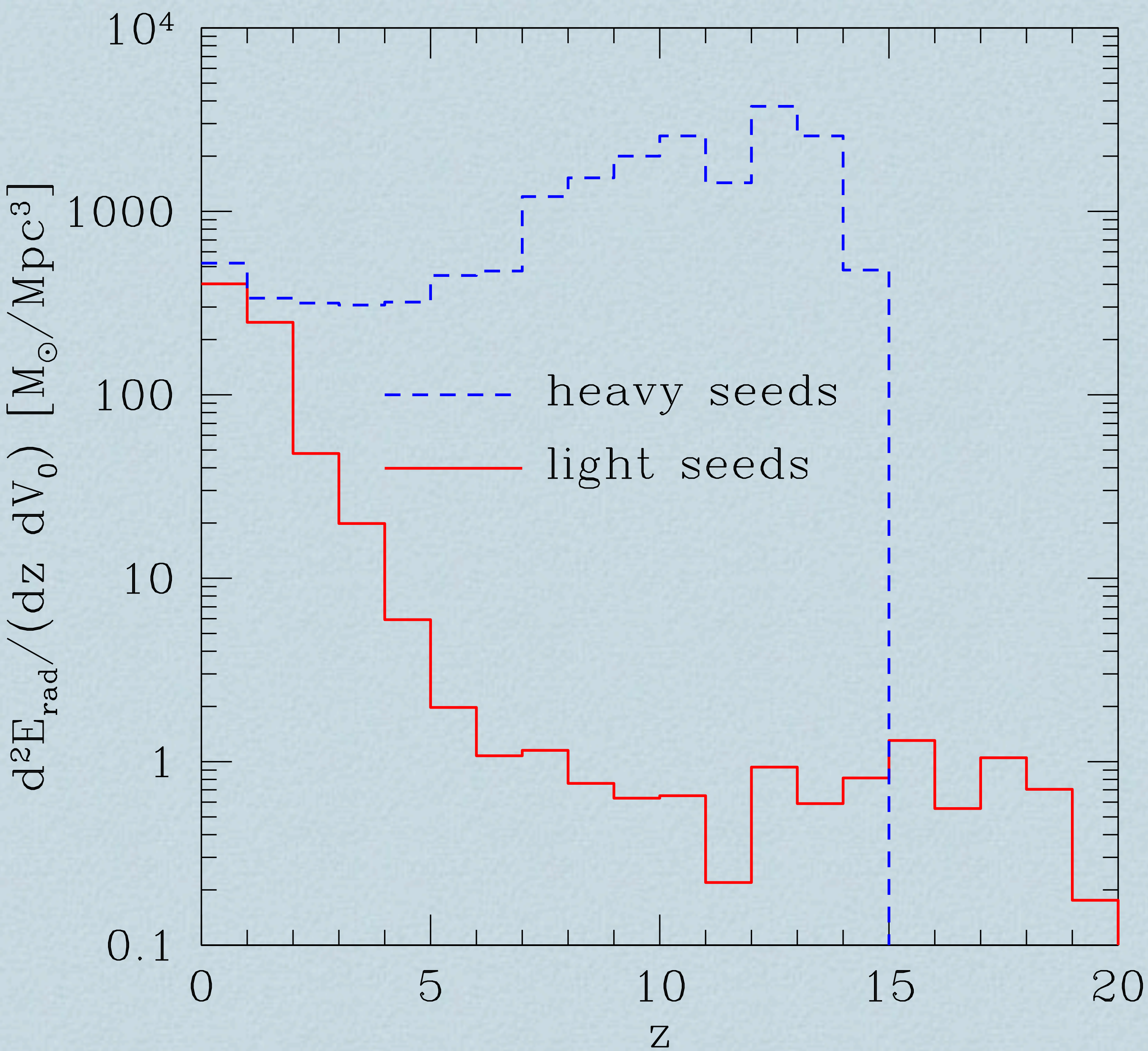
One can see the absence of data for spinning binaries with mass ratio smaller than 1. The inclusion of data in this region would provide a significant check of our formula

The effective-one-body model

- The effective-one-body (EOB) model is a phenomenological model aiming at describing the dynamics and waveforms of BH binaries during the inspiral, merger and ringdown phases, mapping the two body problem onto an effective one body problem, i.e. the motion of a test particle in some effective external metric
- For the accurate calculations of the frequencies and decay times of the quasi-normal modes during the ringdown phase accurate formulae for the final mass and spin of the remnant are required
- Using our formula it is possible to achieve the accumulated phase difference between EOB and numerical waveforms less than 0.05 rad (versus 0.2 obtained with using another old formula for the final mass of the remnant)

Gravitational waves energy emitted by massive BH mergers

- Detectability of massive black hole binary mergers is determined by the energy radiated in gravitational waves per unit comoving volume and as a function of cosmic time



- Using our new formula and the galaxy-formation model of Barausse (2012), we calculated this quantity for two competing models

"light-seed" scenario

$$M_{seed} \sim 100 M_\odot$$

"heavy-seed" scenario

$$M_{seed} \sim 10^5 M_\odot$$

- "Heavy-seed" scenario predicts much stronger GW emission at redshifts $z \gtrsim 3$

Conclusions

- Formula for the final mass of the remnant of black hole binary merger as a function of the mass ratio and initial spins of the black holes is obtained
- The formula is verified by comparison with the large set of numerical data
- Obtained formula is applied to the improvement of effective-one-body approach and the calculation of the energy emitted in gravitational waves

Thank you for your attention