Acceleration of particles by black hole with gravitomagnetic charge immersed in magnetic field

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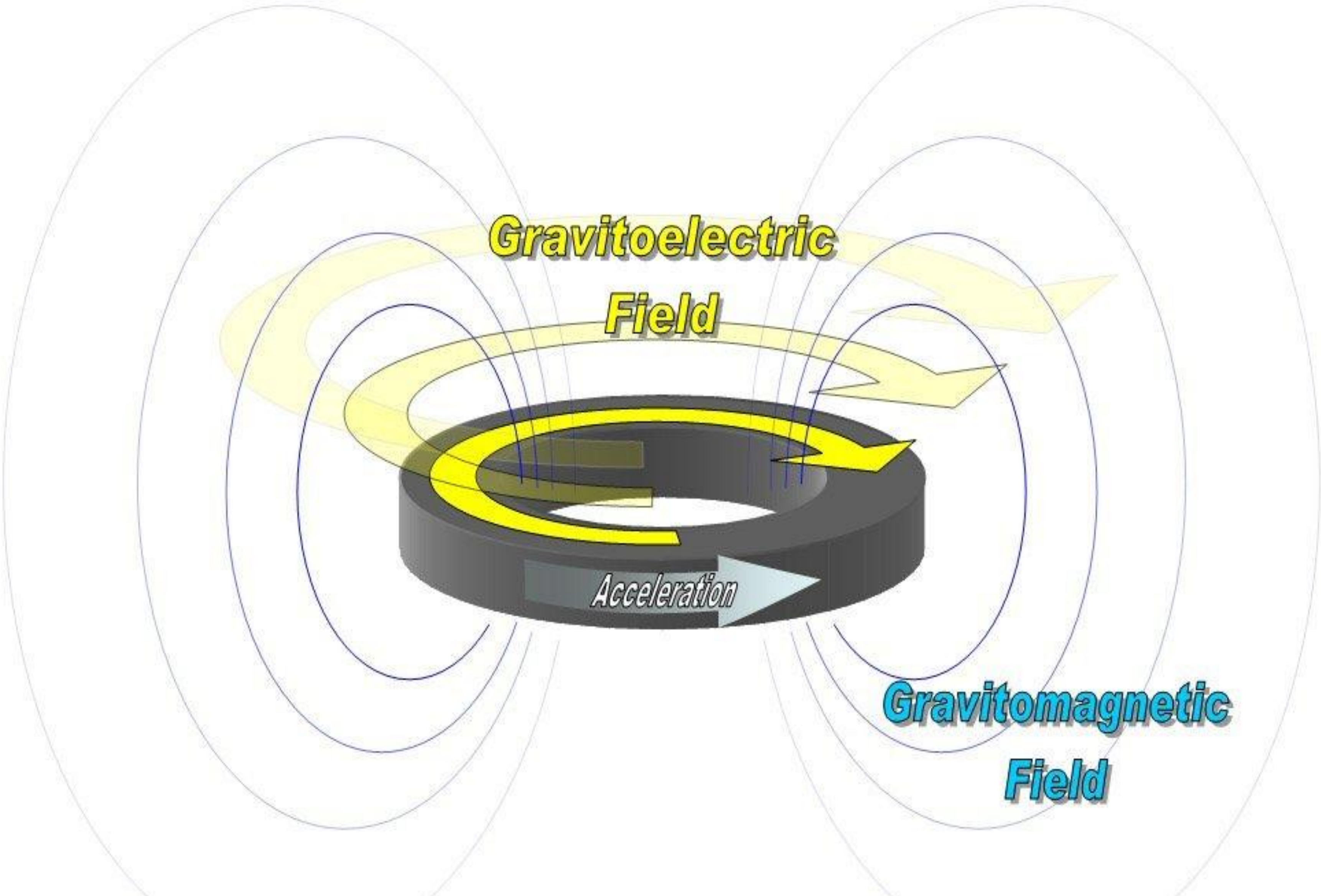
Content


Gravitoelectric
Field
Acceleration
Gravitomagnetic
Field

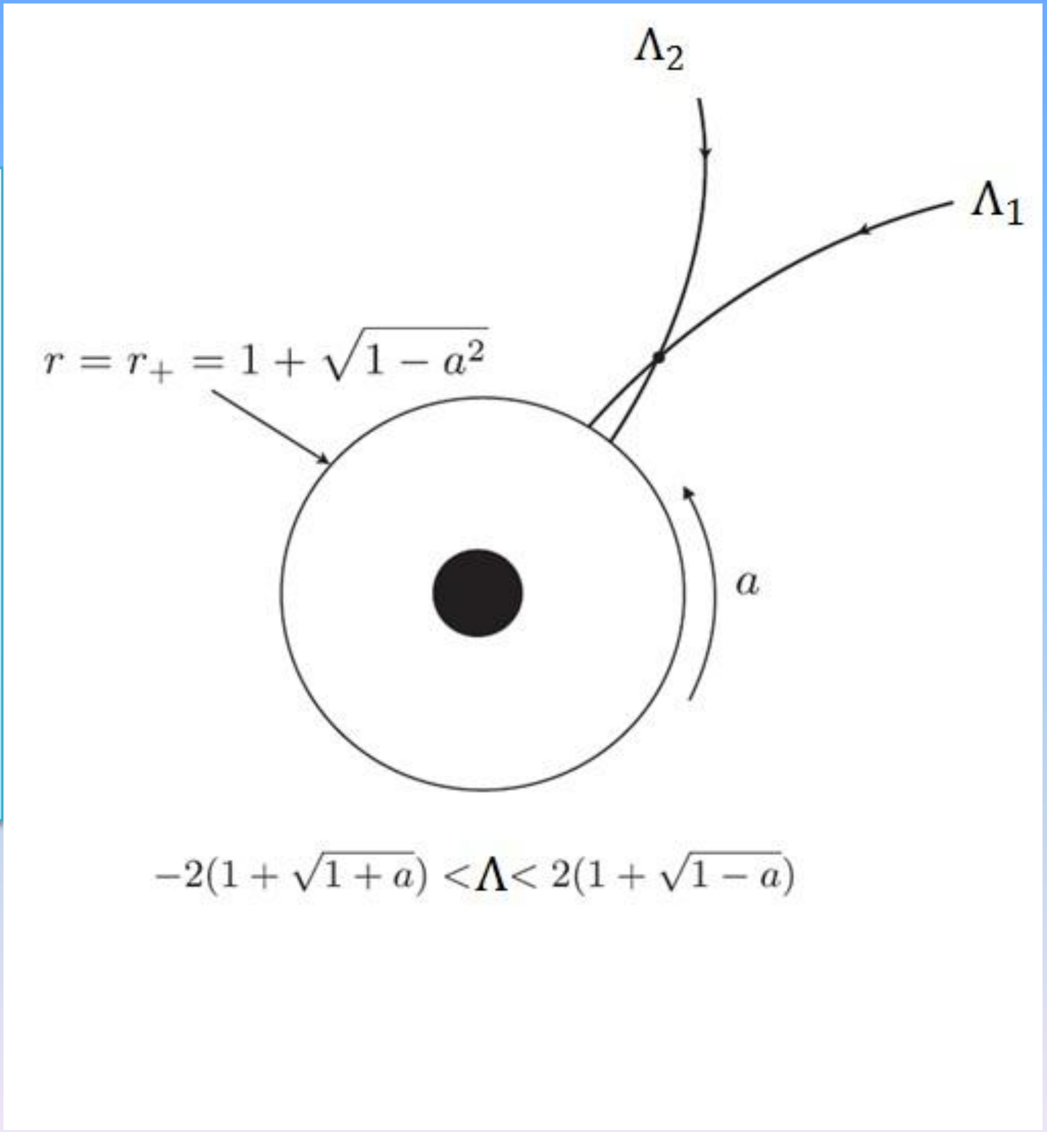
Gravitomagnetic monopole

- At present there is no any observational evidence for the existence of *gravitomagnetic monopole*, i.e. so called *NUT parameter*

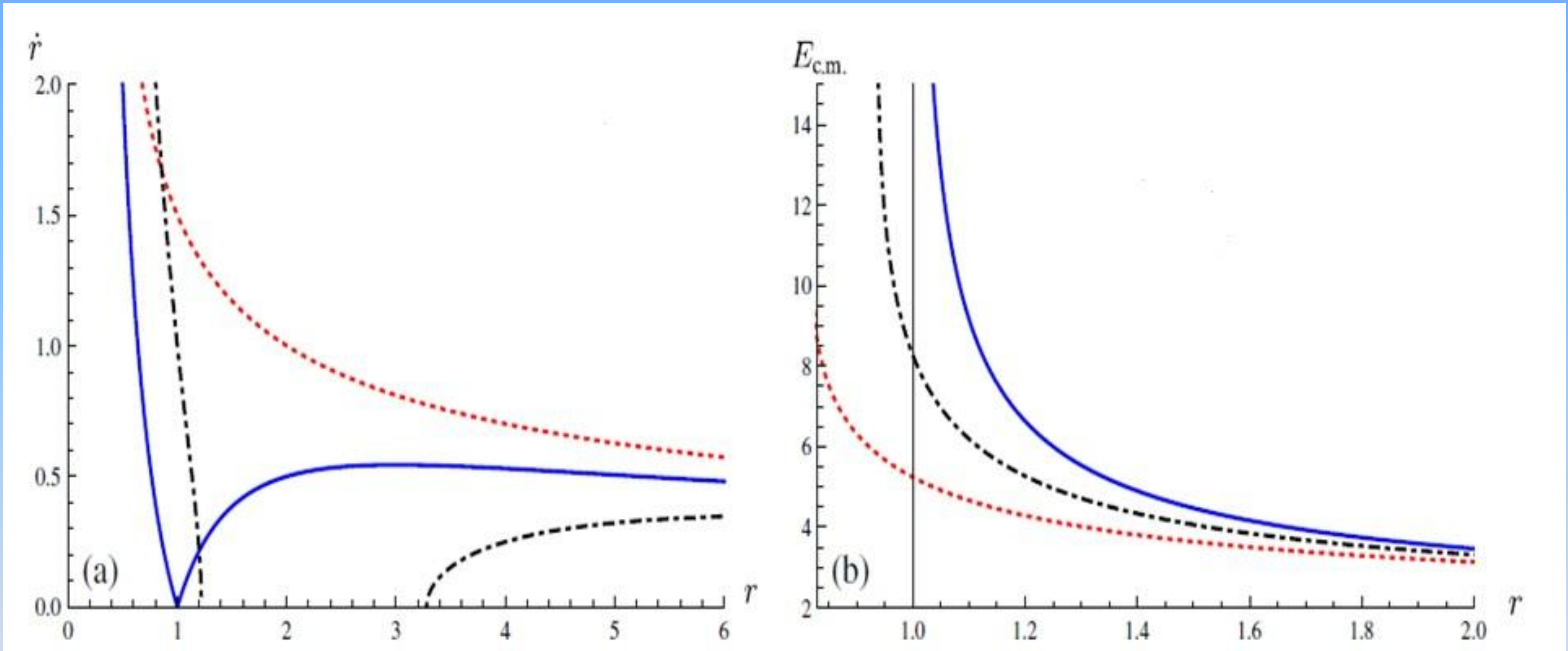
- Therefore study of the motion of the test particles and particle acceleration mechanisms in *NUT spacetime* may provide new tool for studying new important general relativistic effects which are associated with *nondiagonal components of the metric tensor*.

BSW effect

Recently Banados, Silk and West (BSW) demonstrated that collision of particles near an extremely rotating black hole can produce particles of high center-of-mass energy.

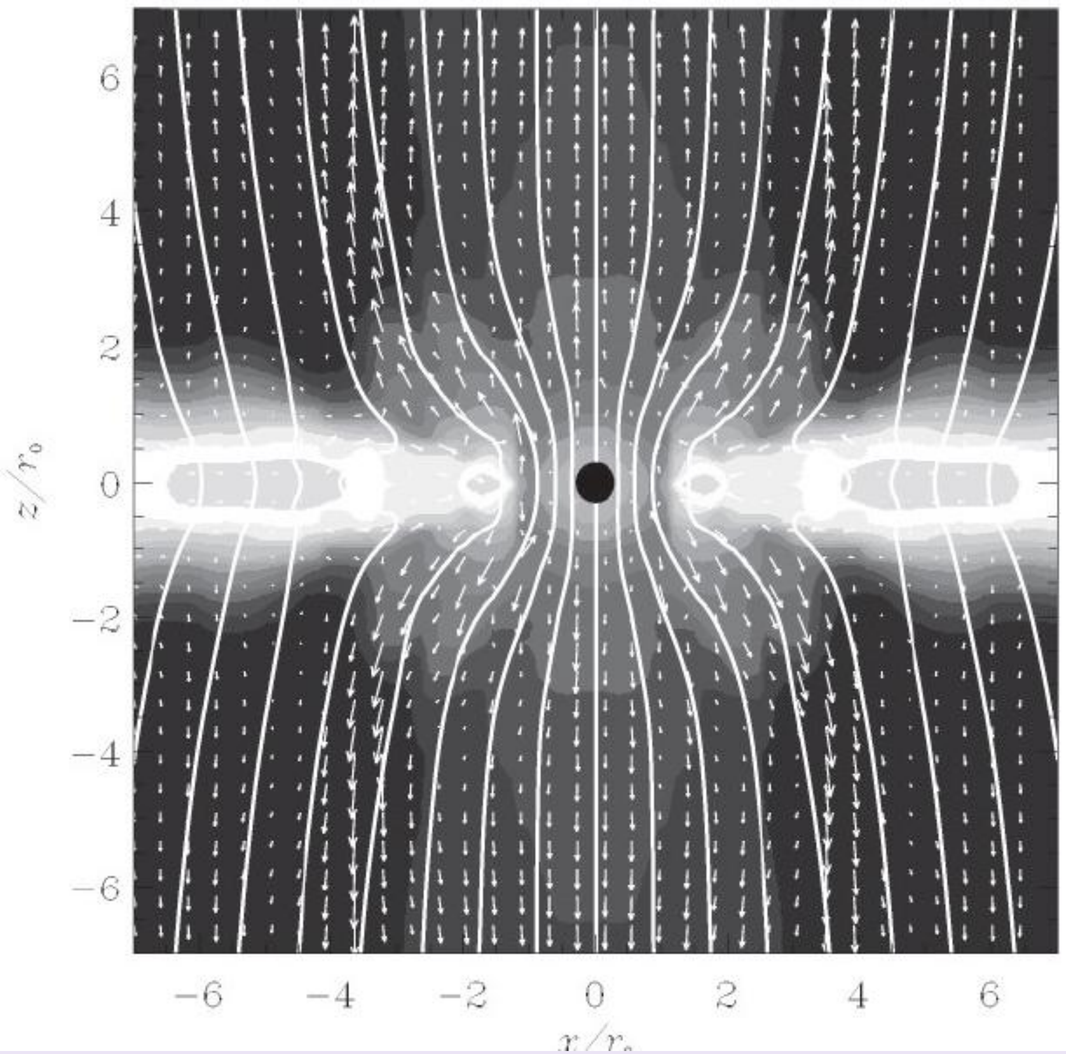


BSW effect



$\dot{r}$
2.0
1.5
1.0
0.5
0.0
(a)
r
0
1
2
3
4
5
6
$E_{\rm c.m.}$
14
12
10
8
6
4
2
(b)
r
1.0
1.2
1.4
1.6
1.8
2.0

BH in external MF



Magnetic field in the BH vicinity acting on a charged particle can change these characteristics significantly

Gravitational effect of a magnetic field

Relative strength of magnetic and gravitational forces acting on a charged particles in the vicinity of the black hole can be characterized by following dimensionless quantity

$$\boxed{b = \frac{qBGM}{2mc^4}}$$

Estimation of the parameter of the magnetic field ***b*** for protons

$$\boxed{b \approx 0.46 \cdot 10^8 \left(\frac{B}{10^8\ Gauss}\right)\left(\frac{M}{10\ M_\odot}\right)}$$

BH in external MF

Maximal collision energy per unit mass for particles close the horizon is

Near Kerr BH

$$\boxed{\frac{E_{cm}}{m} \sim 4.06\,(1-a)^{-1/4}}$$

Near Schwarzschild BH in magnetic field

$$\boxed{\frac{E_{cm}}{m} \sim 1.74\,b^{1/4}}$$

By comparing this relations one can see that they are quite similar if one identifies $\boxed{1-a}$ with $\boxed{b^{-1}}$

Particle motion around a BH

Spacetime metric of the nonrotating BH with gravitomagnetic charge in geometrized unit system

$$\boxed{dS^2 = -\frac{\Delta}{\Sigma}dt^2 - 4\,\frac{\Delta}{\Sigma}l\cos\theta\, dtd\varphi}$$

$$\boxed{+\frac{1}{\Sigma}(\Sigma^2\sin^2\theta - 4\,l^2\cos^2\theta\,\Delta)d\varphi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2}$$

here $\boxed{\Sigma = r^2 + l^2,\ \Delta = r^2 - 2Mr - l^2}$

$\boxed{l}$ - NUT parameter

Equations of charged particle motion

- The 4-vector potential $A^{\alpha} = C_1 \xi^{\alpha}_{(t)} + C_2 \xi^{\alpha}_{(\varphi)}$ of the electromagnetic field will take the following form

$$A_0 = -\frac{\Delta}{\Sigma} Bl \cos\theta \qquad A_3 = \frac{1}{2}\Sigma^2 \sin^2\theta - 2\, l^2 \cos^2\theta\,\Delta$$

- The dynamical equation for a charged particle motion can be written as:

$$m \frac{du^{\mu}}{d\tau} = qF^{\mu}_{\nu}\, u^{\nu}$$

Particle motion around a BH

It is convenient to use two *conserved quantities* associated with the *Killing* vectors: the energy *E > 0* and the generalized azimuthal angular momentum *L*

$$E = -\xi^{\alpha}_{(t)} P_{\alpha} = \frac{\Delta}{\Sigma}\left(m\frac{dt}{d\tau} + 4ml\cos\theta \frac{d\varphi}{d\tau} + qBl\cos\theta \right)$$

$$L = \xi^{\alpha}_{(\varphi)} P_{\alpha} = -4ml\frac{\Delta}{\Sigma}\cos\theta\frac{dt}{d\tau} + \left(\Sigma\, sin^2\theta - 4l^2\frac{\Delta}{\Sigma}\cos^2\theta\right)\left(m\frac{d\varphi}{d\tau} + \frac{qB}{2}\right)$$

where

$$P_{\alpha} = mu_{\alpha} + qA_{\alpha}$$

Equations of charged particle motion

We focus on the motion of the charged particle in the equatorial plane of the black hole, which is orthogonal to the direction of the magnetic field.

$$\frac{dt}{d\tau} = \frac{E}{m}\frac{\Sigma}{\Delta}$$

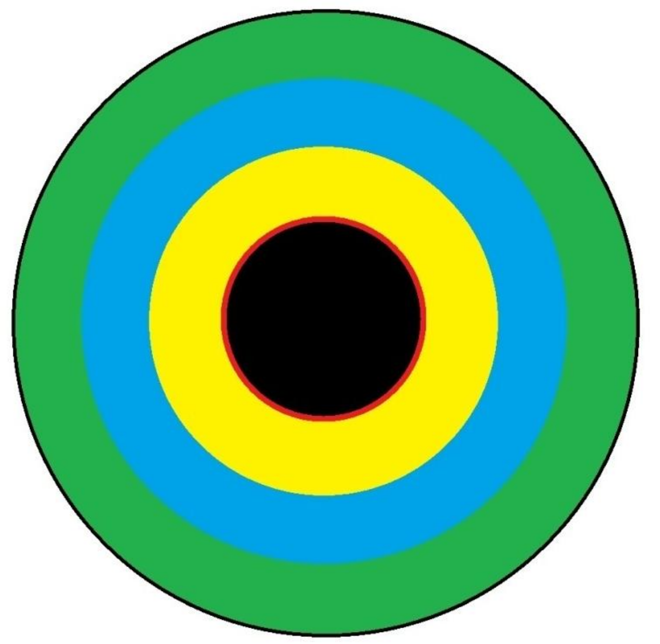
$$\frac{d\varphi}{d\tau} = \frac{L}{m\Sigma} - \frac{qB}{2m}$$

$$\left(\frac{dr}{d\tau}\right)^2 = \left(\frac{E}{m}\right)^2 - U$$

$$U = \frac{\Delta}{\Sigma}(1 + \Sigma\chi^2)$$

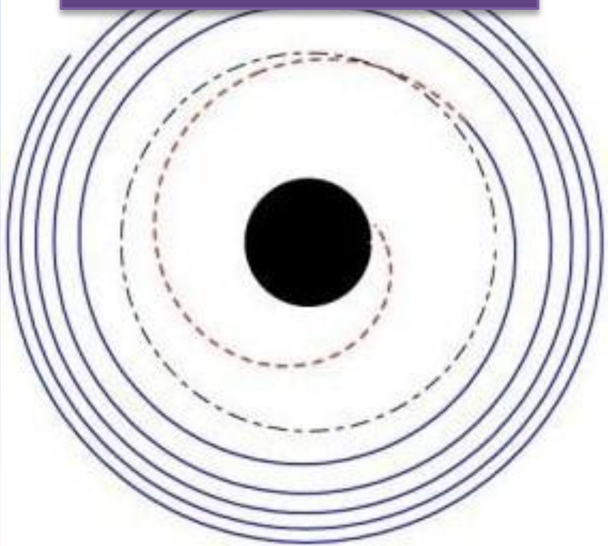
$$\chi = \frac{L}{m\Sigma} - \frac{qB}{2m}$$

Orbits of particle near BH

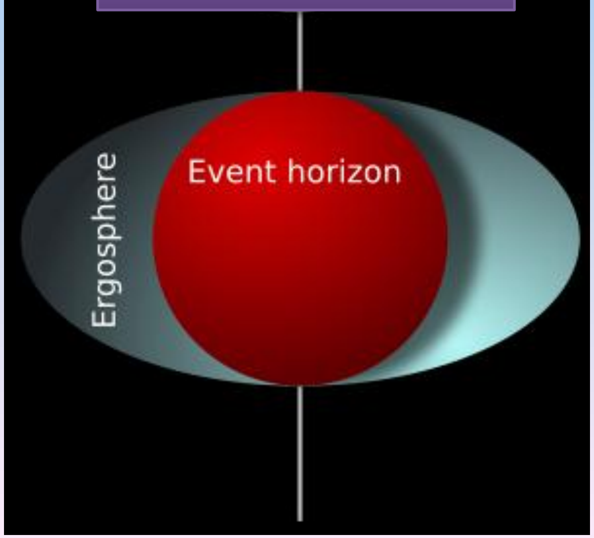


Color	Zone
Green	Stable circular orbits
Blue	Unstable circular orbits
Yellow	No circular orbits
Red line	Horizon
Black	Inside the horizon

Trajectory of particle



Rotating BH



Innermost stable circular orbits

- Position of the *ISCO* radius

$$\boxed{U_{,r} = 0} \qquad \boxed{U_{,rr} = 0}$$

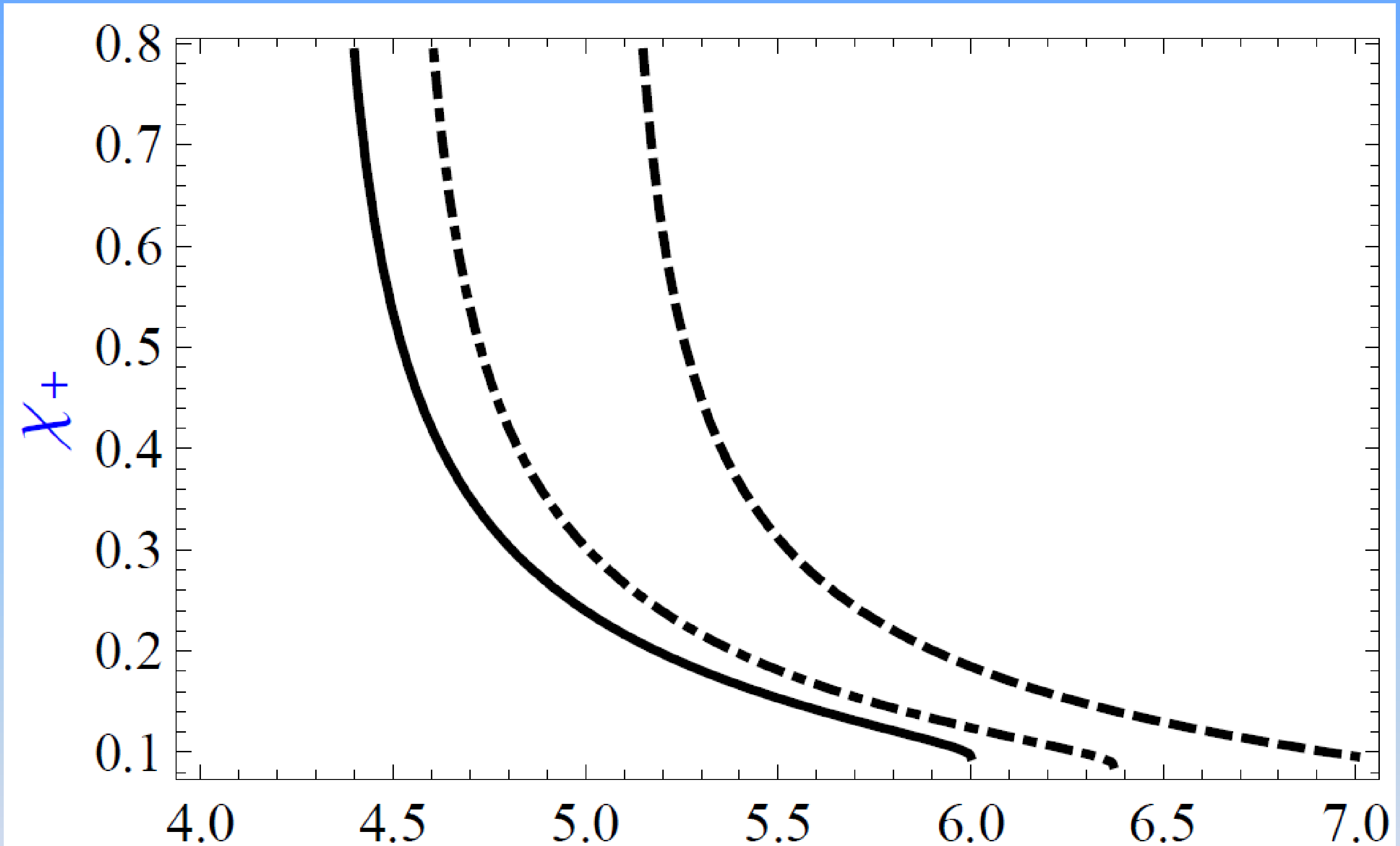
- Expressing given equations by ***r***

$$\boxed{\chi_{\pm} = \pm\left[\frac{K}{\Sigma A}\left(1 \pm \sqrt{1 + \frac{A\Sigma}{K^2}\left(\Delta + \Sigma - \frac{2r^2}{\Sigma}\Delta\right)^2}\right)\right]^{\frac{1}{2}}}$$

$$A = 8\Delta^2 r^2 - 5\Delta^2\Sigma + 12\Delta r^2\Sigma - 8\Delta\Sigma^2 - 3\Sigma^3$$

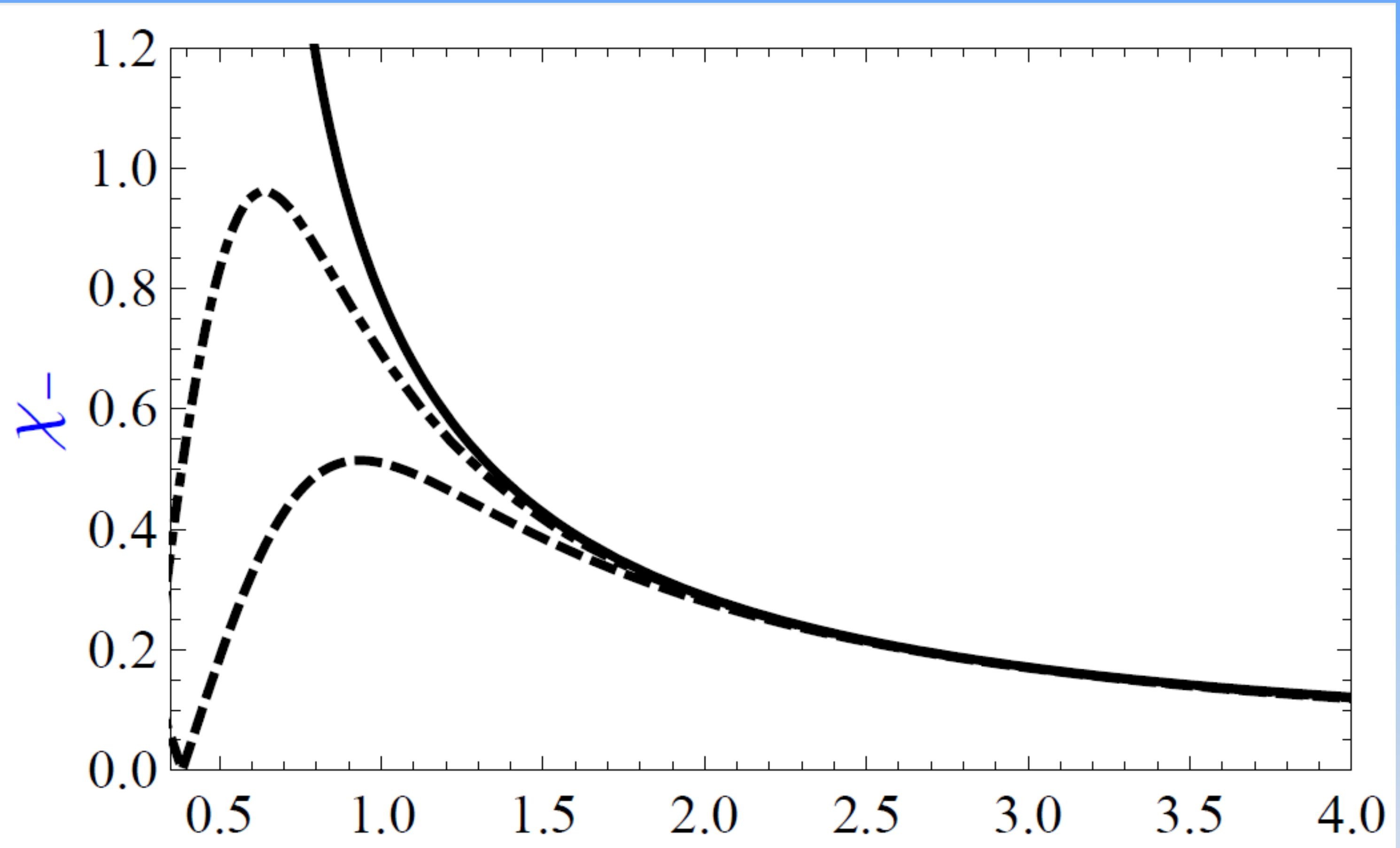
$$K = -2\Delta^2 r^2 + 2\Delta^2\Sigma - 4\Delta r^2\Sigma + 3\Delta\Sigma^2 + \Sigma^3$$

χ_+ as function of radius $\boldsymbol{r}$



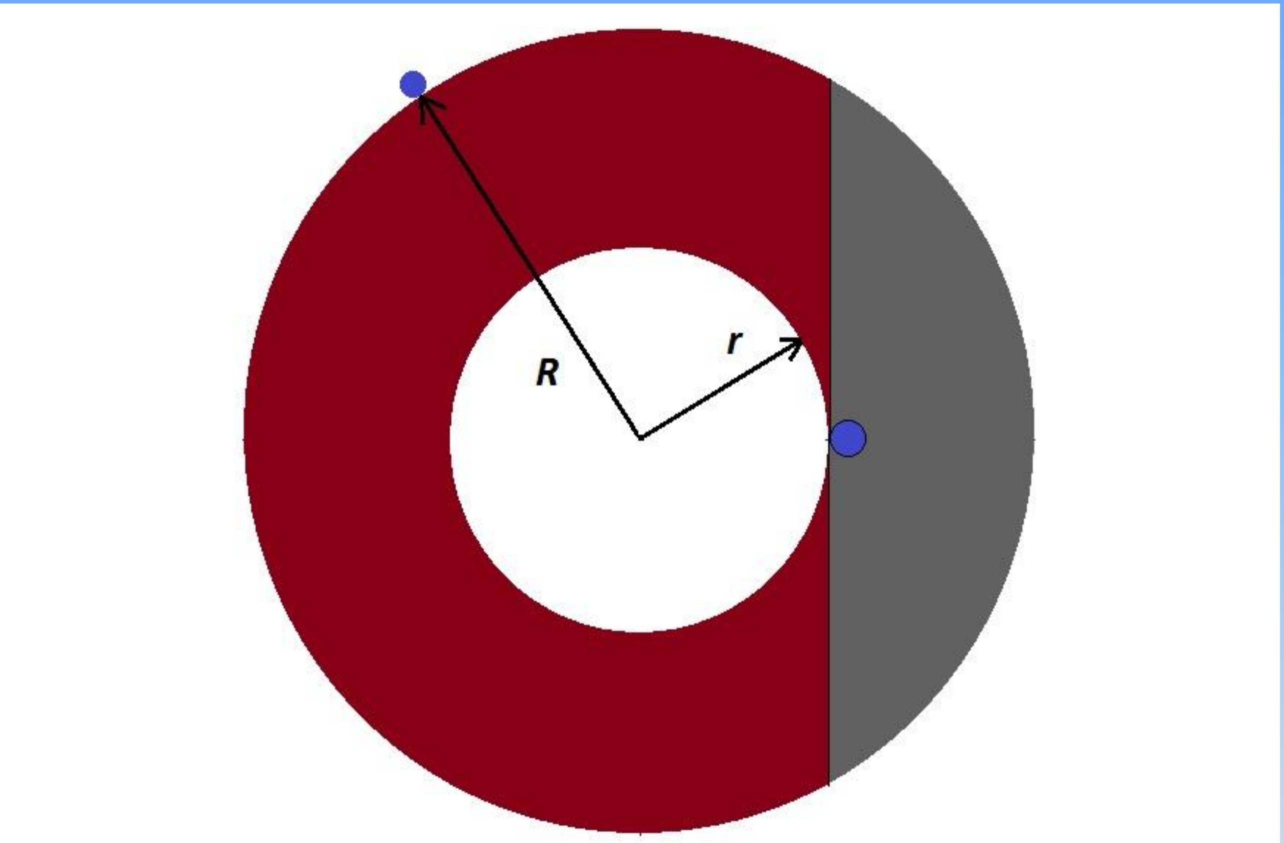
LTR: l=0, l=0.4, l=0.8

χ_- as function of radius *r*



Сверху вниз: *l*=0, *l*=0.4, *l*=0.8

Free fall acceleration as function of distance



$$r > R$$

$$g_0 = G\,\frac{M}{R^2}$$

$$g(r) = g_0\,\frac{R^2}{r^2}$$

$$r < R$$

$$g(r) = \frac{4}{3}\pi\rho G r$$

Lorentz gamma factor

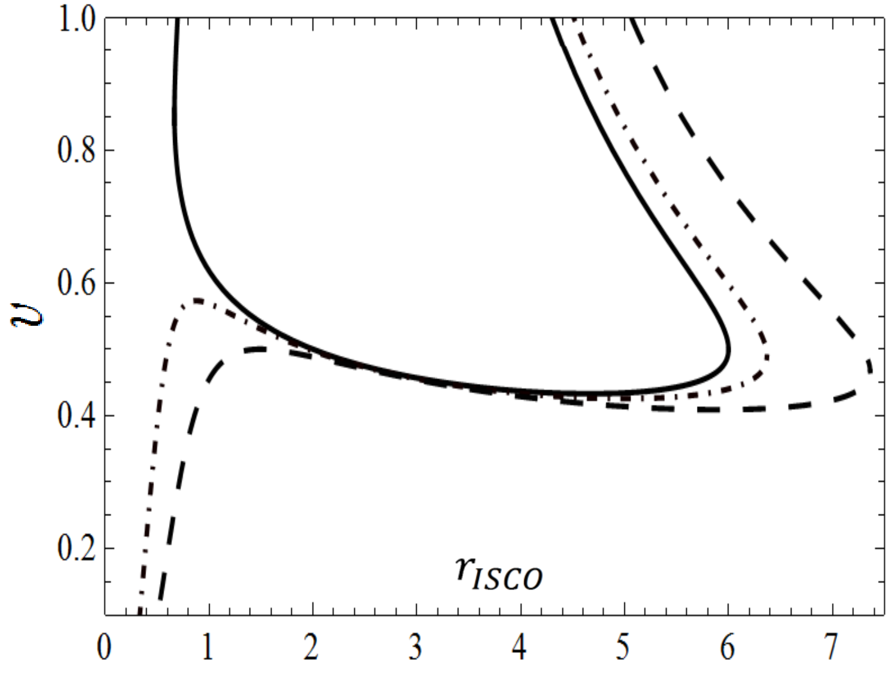
$$\gamma = (1 - v^2)^{-1/2}$$

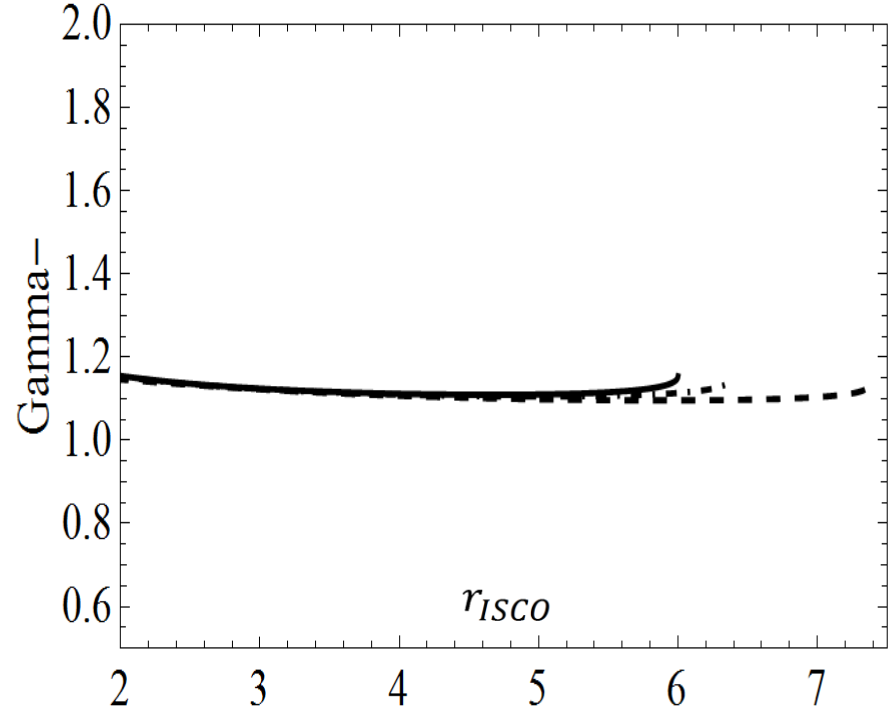
$$\frac{d\varphi}{d\tau} = \frac{v\gamma}{r} = \chi$$

$$\boxed{\gamma^2 = 1 + r^2\chi^2}$$

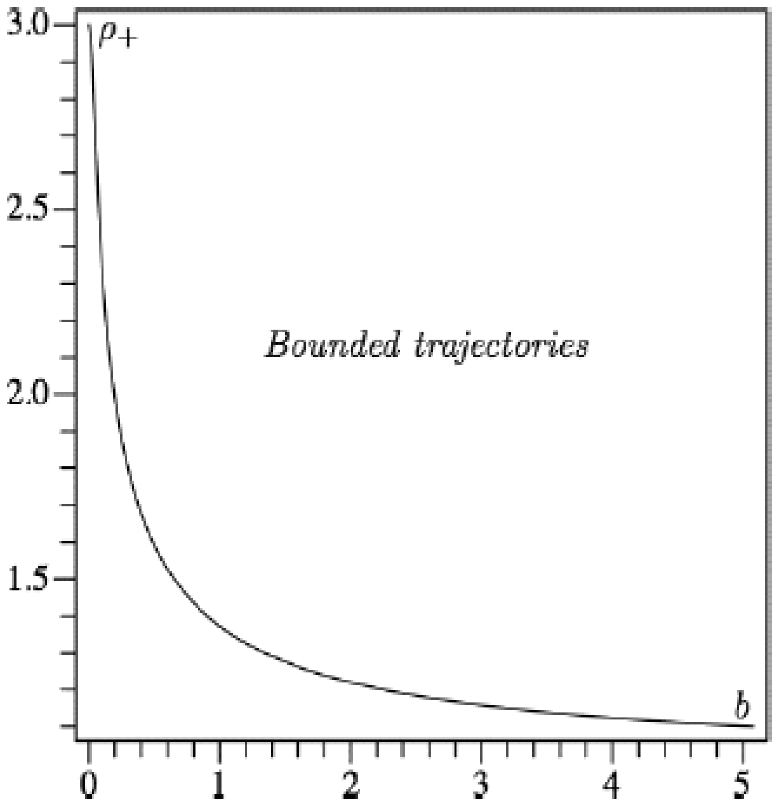
$$\boxed{v = \frac{r\chi}{\sqrt{1 + r\chi}}}$$

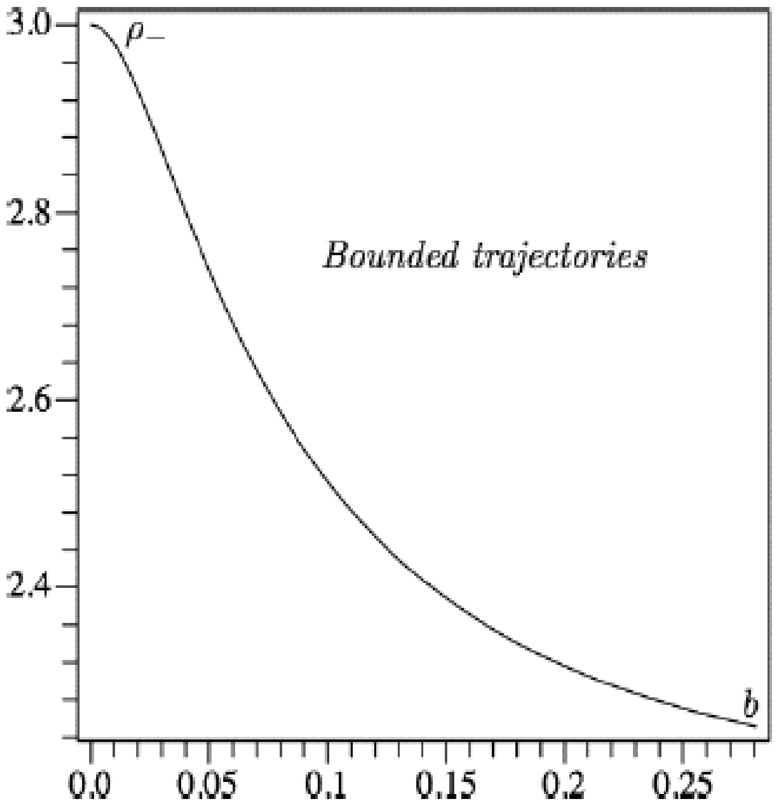
Velocity of a particle and corresponding Gamma-factor at the ISCO





The ISCO radius as function of magnetic field





$$b = qB/2m$$

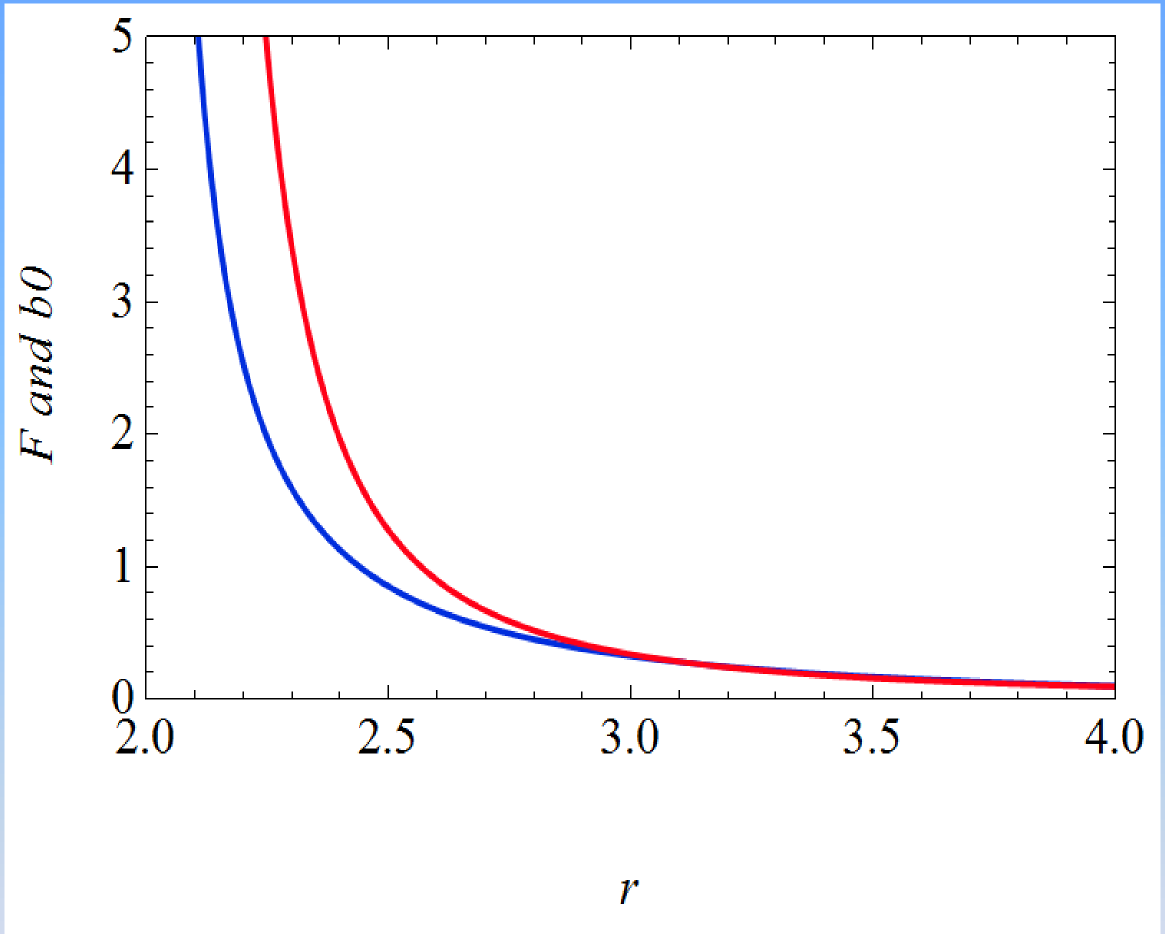
Contribution to the magnetic field on the ISCO

Red line –

function $F(r_{\text{ISCO}})$

Blue line –

b_0 as ***r***



$$b \approx b_0 + F(r_{\text{ISCO}})l^2 + O[l]^3$$

Strong magnetic field on the ISCO

$$\boxed{r \approx r_g + C\, b^{-1} + O[b^{-2}]}$$

$$\boxed{C = \frac{1}{2\sqrt{3}}\sqrt{\frac{(l^2 + {r_g}^2)^3}{(1 + l^2){r_g}^4}}}$$

$l = 0$

$$\boxed{C_0 = \sqrt{\frac{1}{3}}}$$

$l \gg 1$

$$\boxed{C_{max} = \sqrt{\frac{2}{3}}}$$

Particle collision

The center-off-mass energy for a collision of neutral particle with the charged particle at ISCO in exterior magnetic field

$$\frac{E_{cm}}{m} \approx 0.3\sqrt{\frac{96 - l^2}{\sqrt{8 + l^2}}}\, b^{1/4}$$

for $l = 0$ $\quad \frac{E_{cm}}{m} \approx 1.74\, b^{1/4}$

for $l = 0.4$ $\quad \frac{E_{cm}}{m} \approx 1.72\, b^{1/4}$

for $l = 0.8$ $\quad \frac{E_{cm}}{m} \approx 1.70\, b^{1/4}$

$$\frac{E_{cm}}{m} \sim 4.06\,(1 - a)^{-1/4}$$

Conclusion

- We obtained expressions for the energy and angular momentum of the particle in the vicinity of the BH in presence of a gravitomagnetic charge and exterior magnetic field
- Mechanism of the motion of the test particle at ISCO of BH with gravitomagnetic charge immersed in MF was researched
- We obtained expressions for the radii of the ISCO for different values of the NUT-parameter in the approximation of a strong magnetic field
- Influence of the NUT-parameter on the energy of the particle collision in the vicinity of the BH immersed in magnetic field was found
- It was shown that presence of the nonvanishing NUT-parameter decrease the maximal collision energy of particles in the vicinity of BH immersed in magnetic field

Thank You