Excitations of Global Oscillations in BH Accretion Disks and the Origin of HFQPOs

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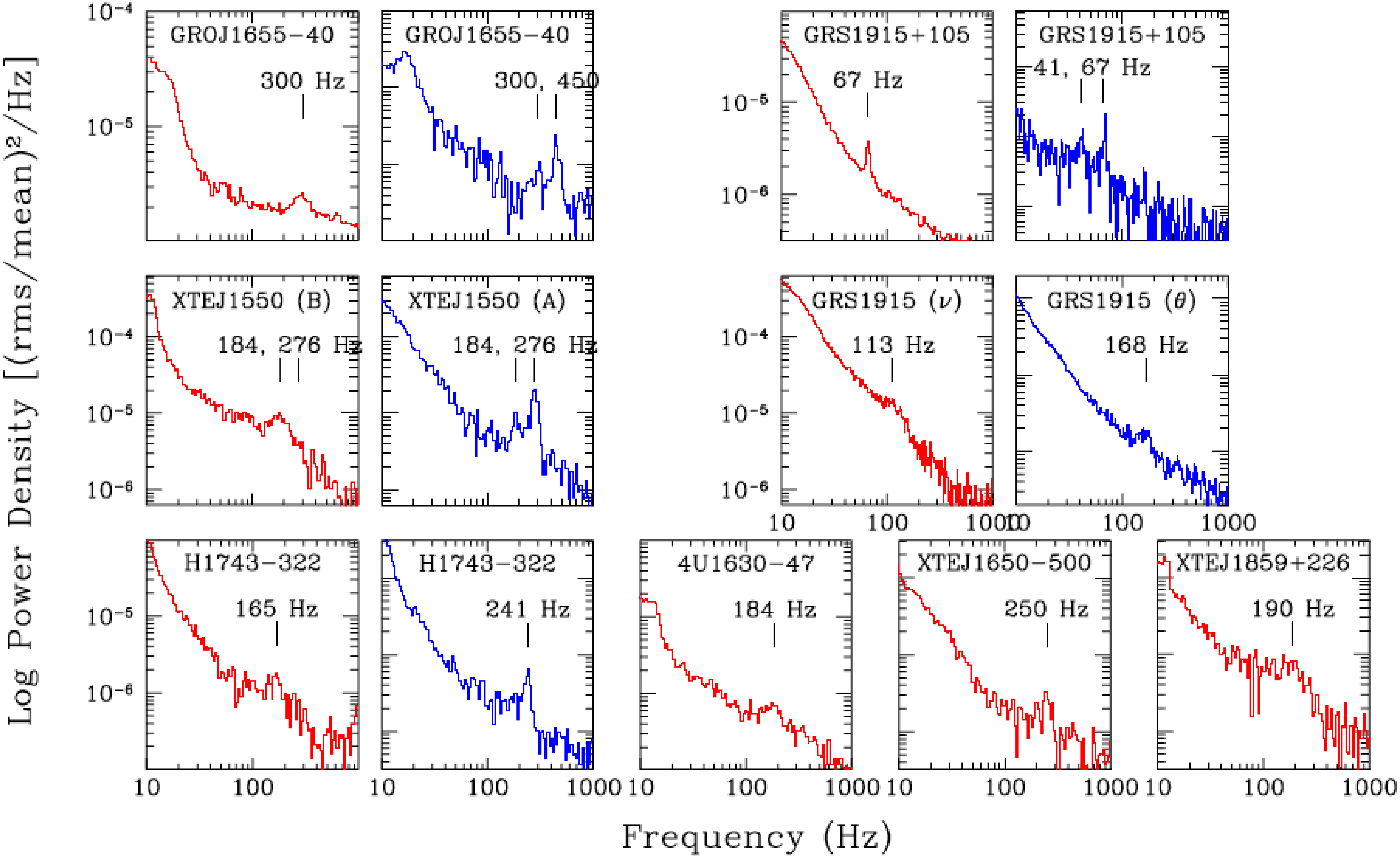
Black Hole Astrophysics, Winchester, UK, 7/21/2011

High-Frequency QPOs in BH X-Ray Binaries



Remillard & McClintock 2006

Basic Facts about HFQPOs (for a naïve theorist)

- 40-450 Hz: ~ orbital frequency at r_{isco}
- Frequency stable (<10% change when Mdot doubles)
- Some systems: ~2:3 ratio
- Weak QPOs: ~1% flux variation (in hard X-rays), Q~2-10
- Only occur in "Transitional state" (="very high state") (Episodic jet)

Phenomenology will be helped by LOFT…

Ideas/Models of HFQPOs

- Orbiting blobs (hot spots) in disks (Stella et al ‘99; Schnittman & Bertschinger ‘04)
- Nonlinear resonances of some kind (Abramowicz, Kluzniak, Horak, Rebusco)
- Acoustic modes in torus (Rezzolla el al ‘03; Lee, Abramowicz & Kluzniak ‘04; Blaes et al. ‘07; Sramkova et al ‘07; Horak’ 08)
- Disk/Magnetosphere Boundary Layer Oscilations (Li & Narayan ‘04; Tsang & DL ‘09)

- Oscillation modes in relativistic disks (Kato; Wagoner & collaborators)
  - -- m=0 inertial modes excited by global disk deformation (e.g. warps) (Kato ‘03,’ 08; Ferreira & Ogilvie ‘08; Henisey et al.10)
  - -- Rossby modes trapped in special region of a magnetic disk (Tagger & Varniere ‘06; see also Tagger & Pallet ‘99; Varniere & Tagger’ 02)
  - **-- Cornell effort:** Mode growth due to corotational resonance, magnetic fields (DL & Tsang ‘09; Tsang & DL ‘08,’ 09a,b; Fu & DL ‘09,’ 11a,b)

Main points of this talk:

P-modes ("inertial-acoustic modes", "spiral density modes")

--Trapped (partially) in the innermost region of disk

-- Frequencies can be calculated: robust, agree with observations

-- Can grow due to corotation resonance ("corotational instability")

GR plays an important role

-- Large-scale B field may enhance the mode growth

(=>"transitional state")

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References: DL & Tsang 2009; Tsang & DL 2008, 2009; Fu & DL 2009,2011
Fu & DL 2012; Horak & DL 2012: in prep

Waves in 2D disks (Spiral density waves):

$$\delta v, \delta\Sigma \propto \exp(im\varphi - i\omega t)$$

Can propagate only in the region:

$$r < r_{\rm ILR} \quad \text{or} \quad r > r_{\rm OLR}$$

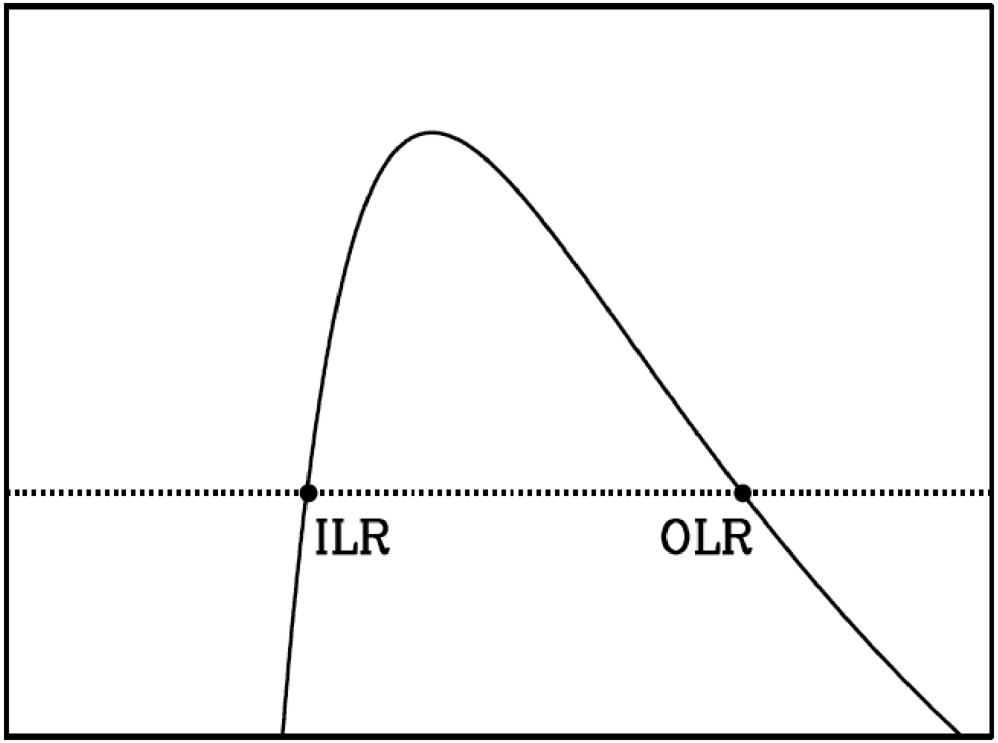
Lindblad Resonances: $\omega - m\Omega(r) = \pm\kappa(r)$

where $\Omega(r) = \text{disk rotation rate}$

$\kappa(r) = \text{radial epicyclic frequency}$

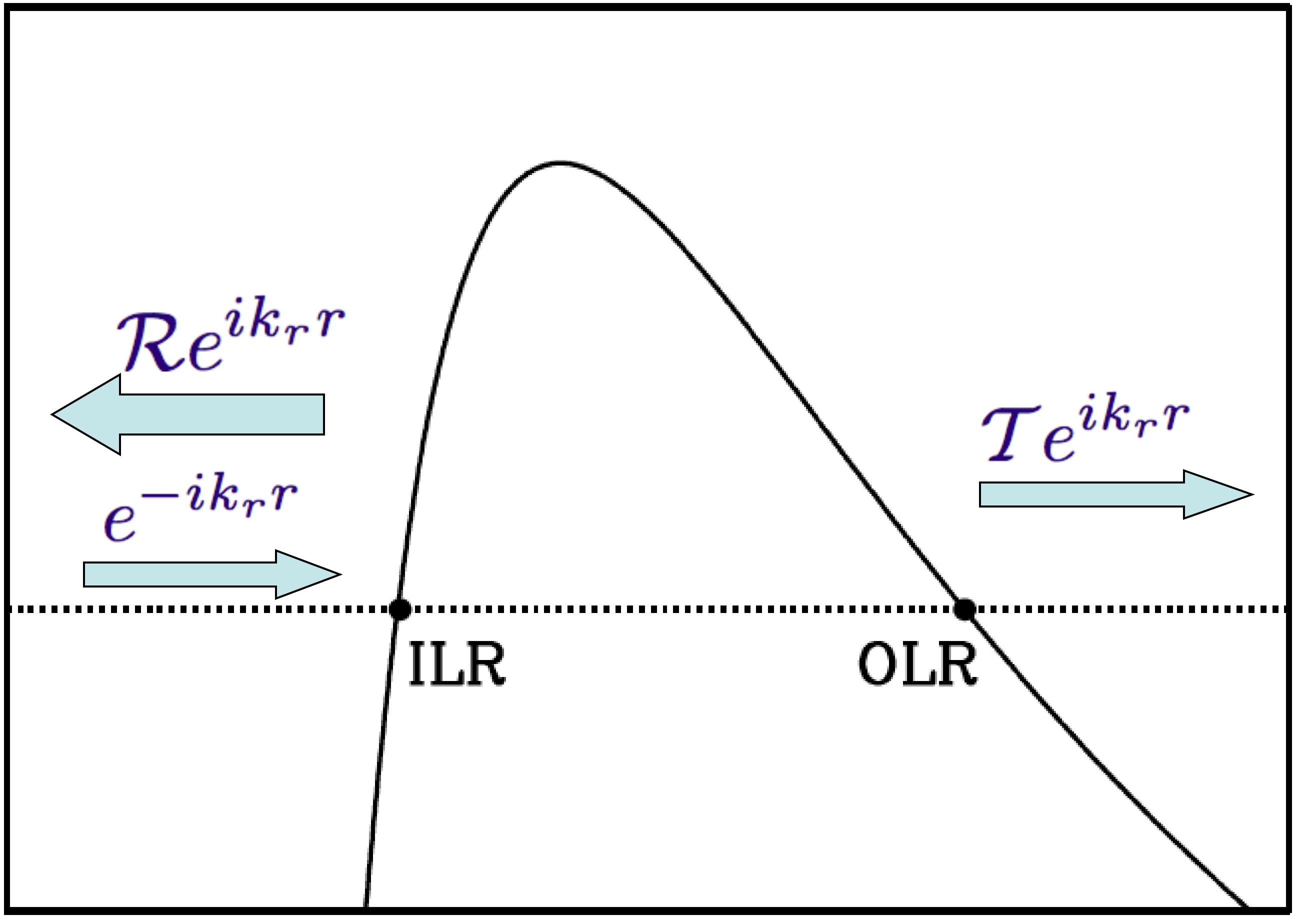
$$\kappa^2 = \frac{2\Omega}{r}\frac{d}{dr}(r^2\Omega)$$

Wave propagation diagram (effective potential)



wave at $r > r_{\mathrm{OLR}}$: $\omega/m > \Omega \ \Rightarrow$ positive energy

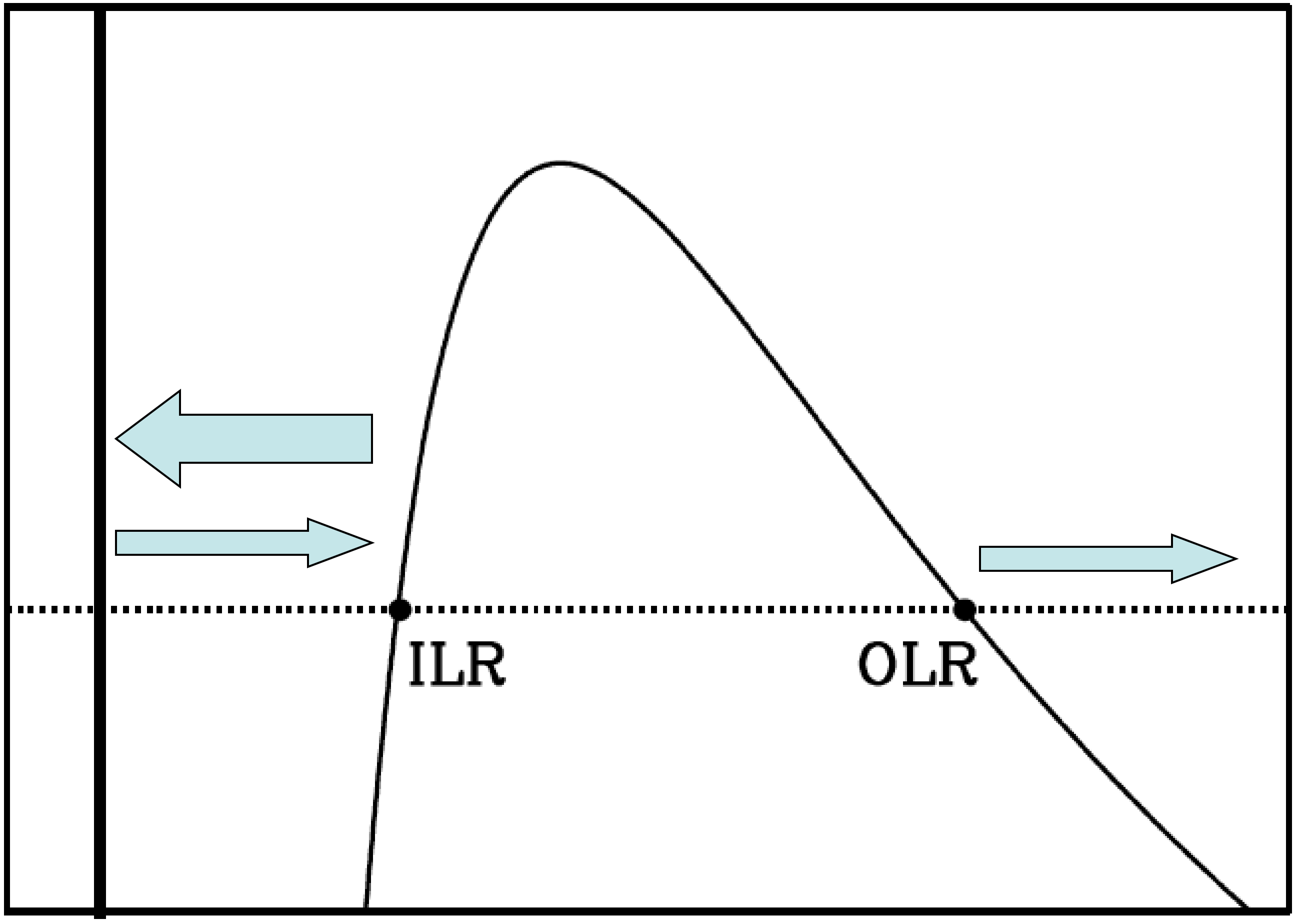
wave at $r < r_{\mathrm{ILR}}$: $\omega/m < \Omega \ \Rightarrow$ negative energy



$$(-1) = (-1)|\mathcal{R}|^2 + |\mathcal{T}|^2$$

$$\Rightarrow \ |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 > 1$$

Super-reflection

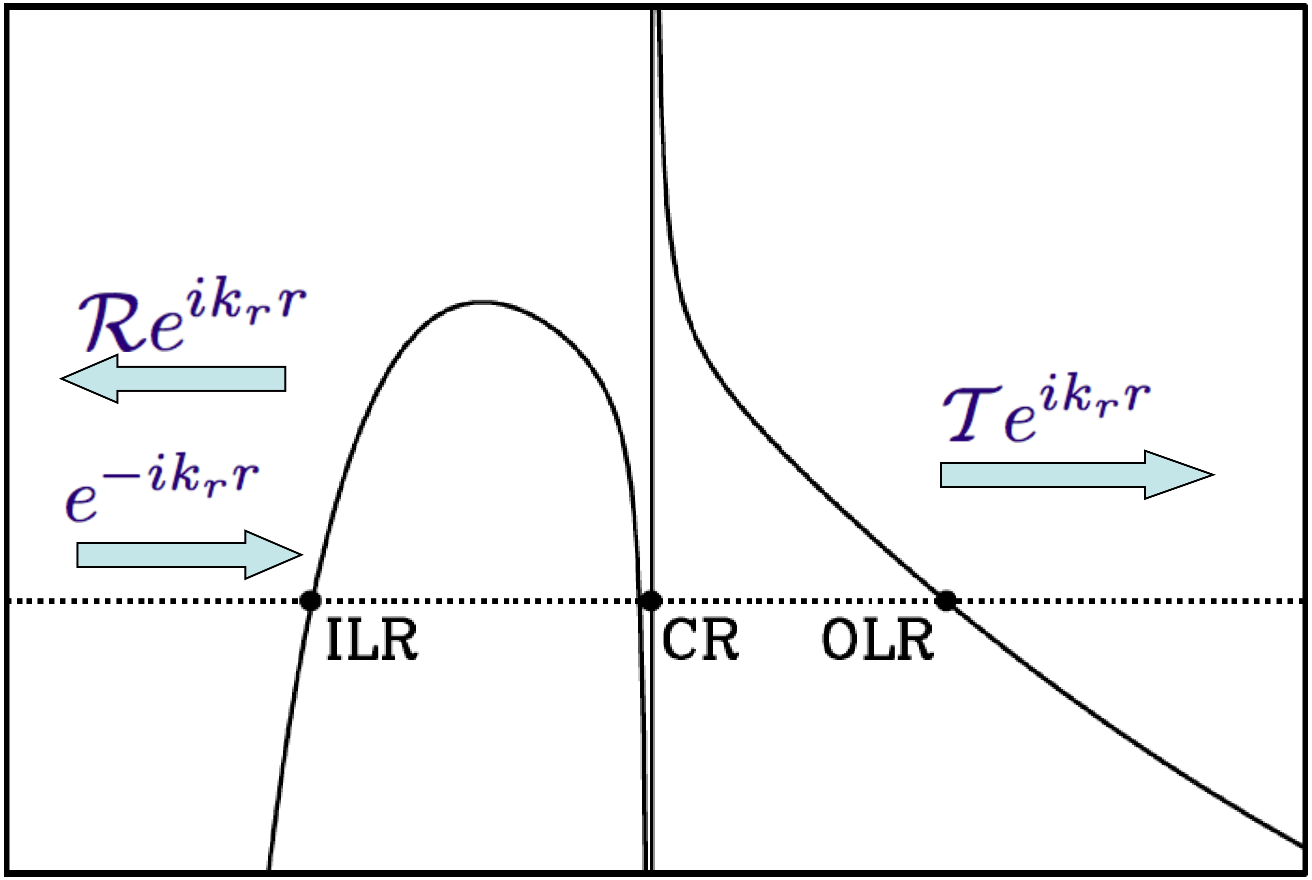


Trapped mode between r_{in} and r_{ILR} : overstable

Even more interesting…

Corotation resonance, where

$$\omega/m = \Omega$$



$$(-1) = (-1)|\mathcal{R}|^2 + |\mathcal{T}|^2 + \mathcal{D}_c$$

$$\Rightarrow |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + \mathcal{D}_c$$

Wave absorption at corotation

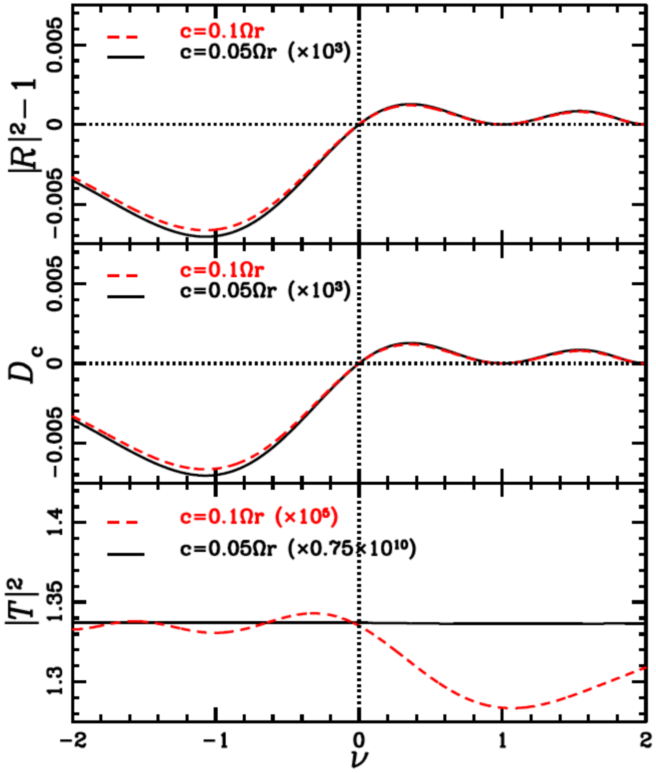
Can have both signs !

Calculations of reflectivity/transmission:

$$\delta h = \sqrt{S/k}\left[\exp\left(-i\int_{r_{\rm IL}}^{r} kdr + \frac{\pi}{4}\right) + \mathcal{R}\exp\left(i\int_{r_{\rm IL}}^{r} kdr - \frac{\pi}{4}\right)\right]$$

$$\delta h = \sqrt{S/k}\,\mathcal{T}\exp\left(i\int_{r_{\rm OL}}^{r} kdr + \frac{\pi}{4}\right)$$

- Solve wave equation in different regions
- Match the solutions using asymptotic expansions
- Around corotation: Whittaker function; Stokes phenomenon



Tsang & DL

$$\mathcal{R} = \frac{1+\frac{1}{4}\left(e^{-i2\pi\nu}+\sin^2\pi\nu\right)e^{-2\Theta_{\rm II}} + \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIa}}}{(\Gamma(1-\nu))^2} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIb}}}{(\Gamma(1+\nu))^2}}{1-\frac{1}{4}\left(e^{-i2\pi\nu}+\sin^2\pi\nu\right)e^{-2\Theta_{\rm II}} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIa}}}{(\Gamma(1-\nu))^2} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIb}}}{(\Gamma(1+\nu))^2}}$$

$$\mathcal{T} = \frac{ie^{-2\Theta_{\rm II}}e^{i\pi\nu}}{1-\frac{1}{4}\left(e^{-i2\pi\nu}+\sin^2\pi\nu\right)e^{-2\Theta_{\rm II}} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIa}}}{(\Gamma(1-\nu))^2} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIb}}}{(\Gamma(1+\nu))^2}}$$

$$\Theta_{\rm IIa} = \int_{r_{\rm IL}}^{r_c} |k|\,dr \qquad \Theta_{\rm IIb} = \int_{r_c}^{r_{\rm OL}} |k|\,dr$$

Reflectivity at ILR: $|\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + \mathcal{D}_c \simeq 1 + \mathcal{D}_c$

Sign depends on sign of $d\zeta/dr$

$$\zeta = \frac{\kappa^2}{2\Omega\Sigma}$$ (vortensity)

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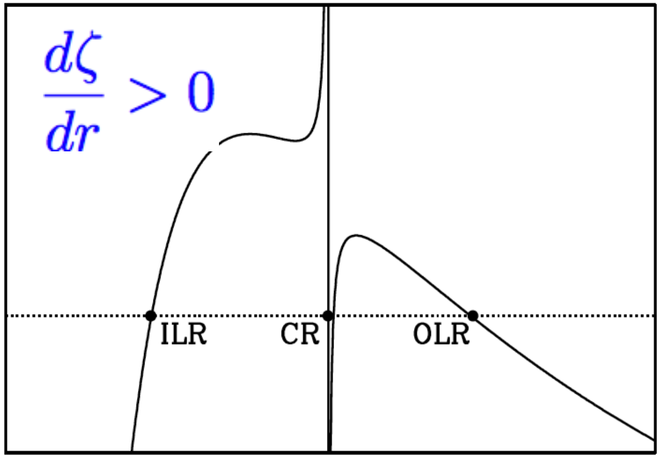
(vortensity)



$$\Rightarrow \mathcal{D}_c > 0$$

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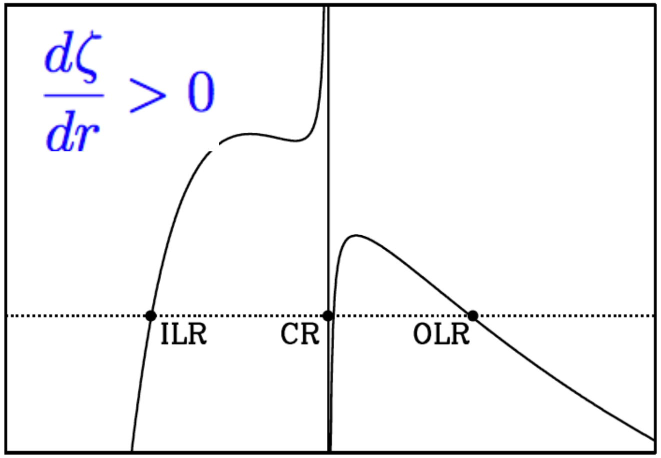

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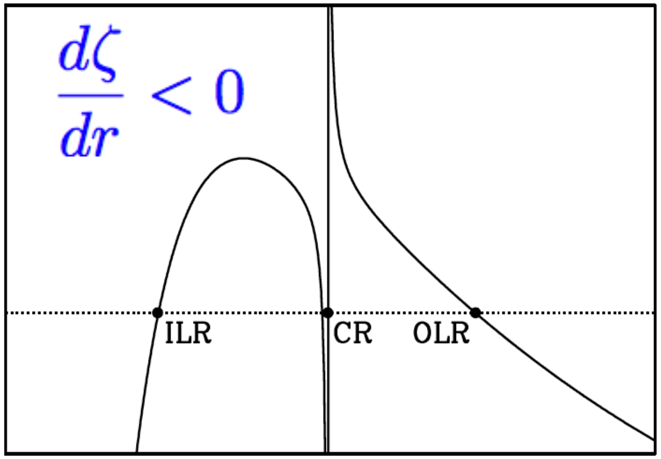

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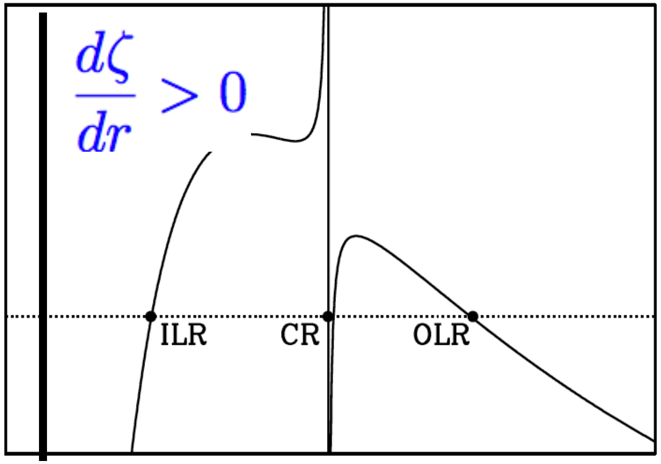

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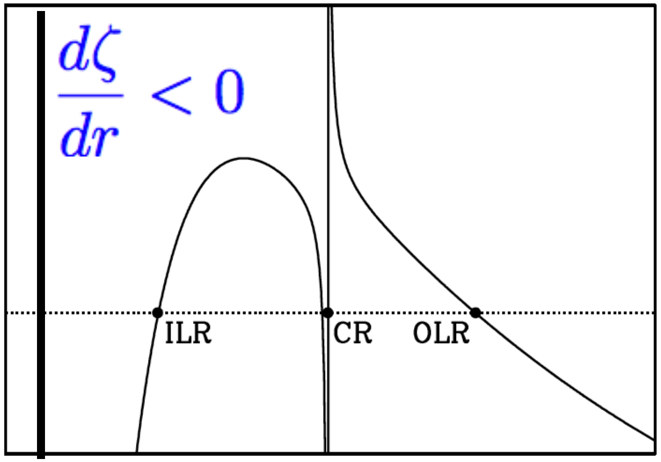
Overstable mode

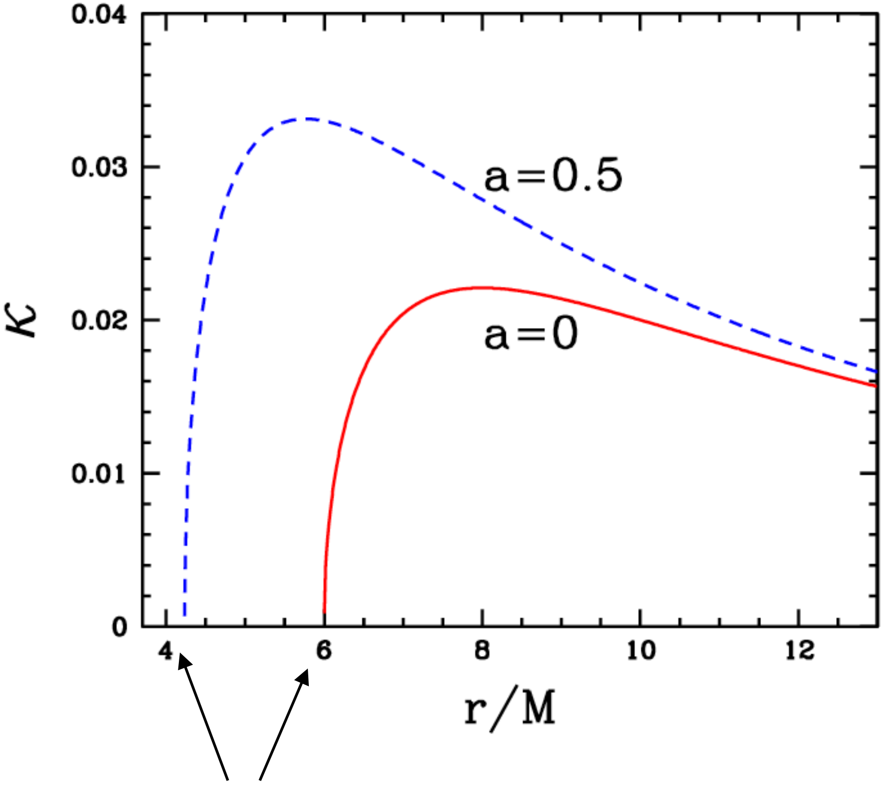


$\Rightarrow \mathcal{D}_c < 0$

Damped mode

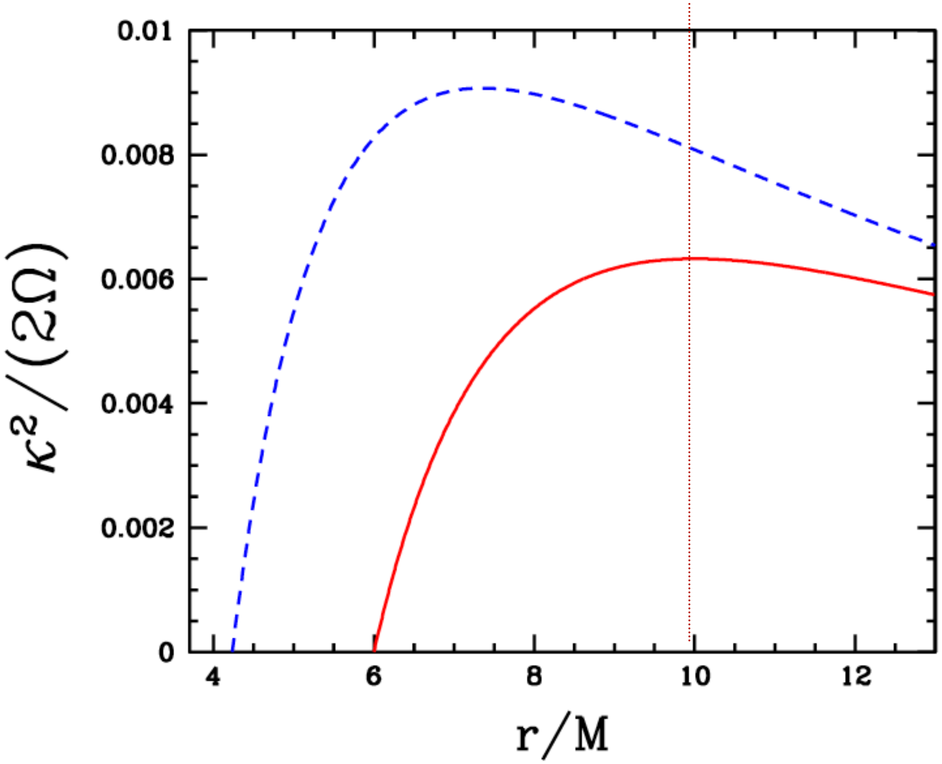
General Relativity Effect

GR makes $d\zeta/dr > 0$
in the Inner-most disk region
==> makes the mode grow !

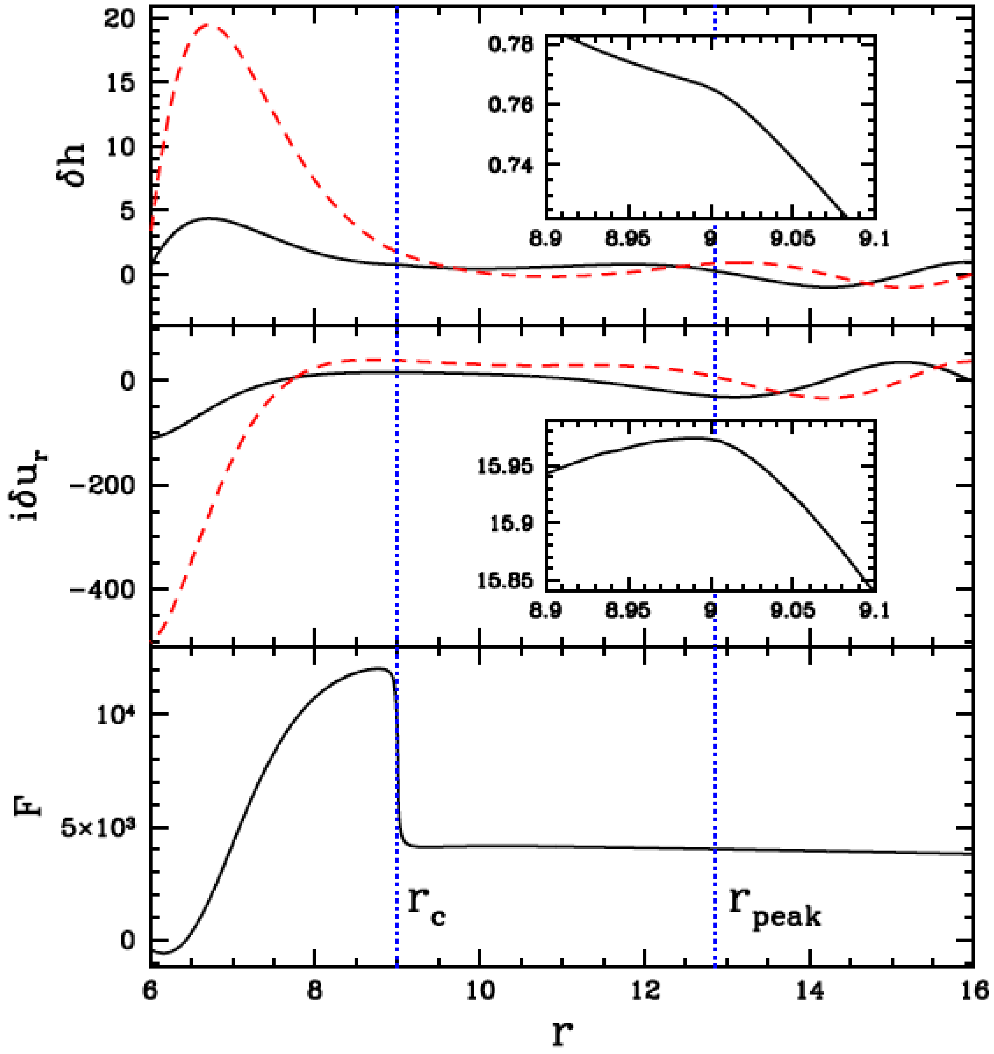


ISCO

Vortensity $\zeta = \frac{\kappa^2}{2\Omega\Sigma}$



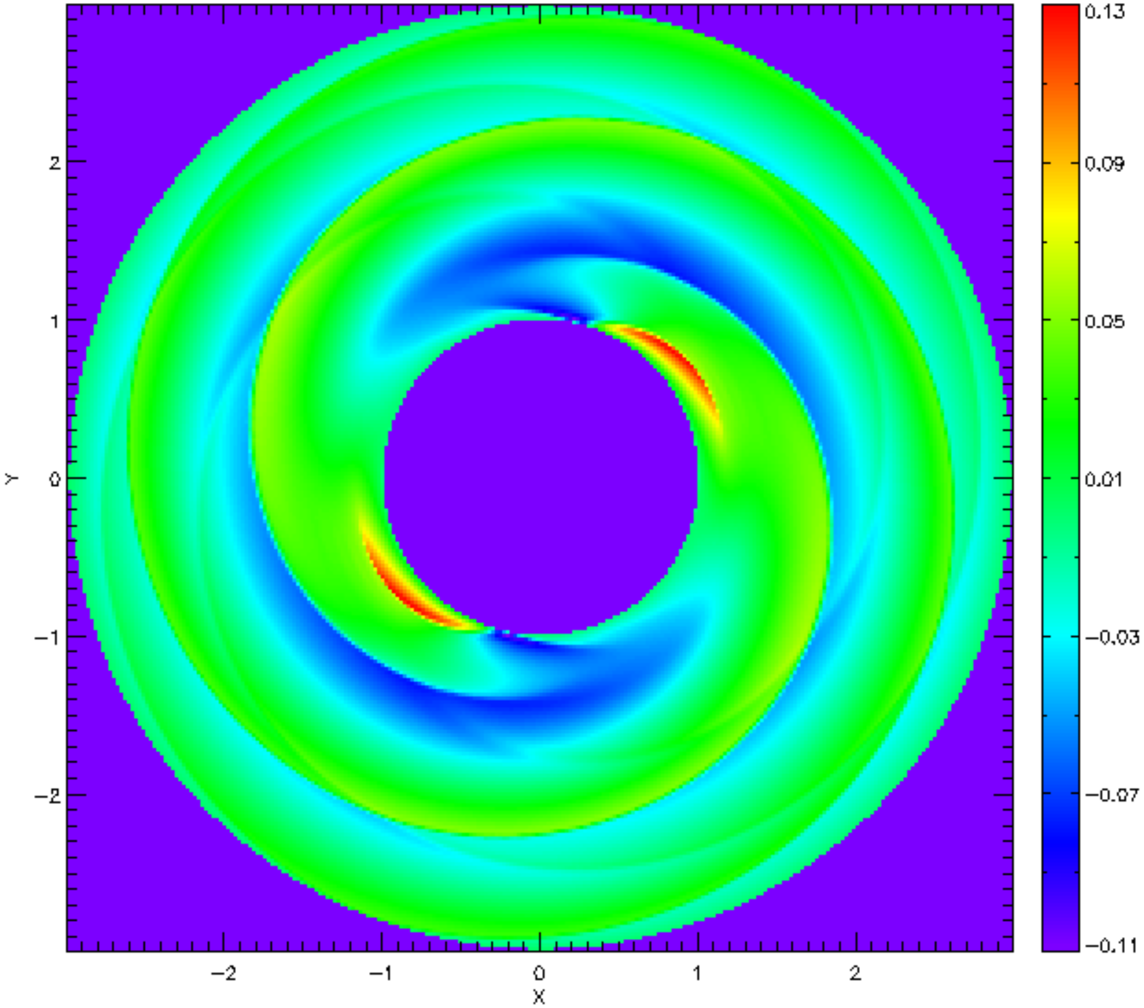
Linear Mode Calculation (Mode freq. and growth rate)



$$\Sigma \propto r^{-1}, \quad c_s = 0.1 r\Omega, \quad m = 2$$
$$\omega_r = 0.93\,\Omega_{\rm ISCO}, \quad \omega_i/\omega_r = 0.0029$$

DL & Tsang 2009,2010

Nonlinear Simulation (2D) of Growing Modes



Wen Fu & DL 2011, in prep

Properties of Overstable Disk P-Modes:

Low-order p-modes trapped between inner disk edge and ILR

$$\omega \simeq \beta m \Omega(r_{\rm in})$$

$\beta = 0.55\text{–}0.75$ depending on disk models and inner BC

- Mode frequency varies (~10%) as Mdot changes (~3)
- Mode frequency consistent with known BH mass (and spin)
- Frequency ratio approximately: 1:2:3:4… (not exactly)

Grow due to corotation resonance (GR plays important role)

- Mode growth rate: m=1 much slower than m=2,3,4…
  ==> Most visible modes are m=2,3

A promising candidate for HFQPOs

Complications…

Mode damping due to radial infall

For standard thin (SS) disks, radial velocity incresases rapidly near $r_{ISCO.}$

==> Tends to damp the p-modes;
(but not completely, due to sharp density gradient around r_{in})

Competition: mode growth (due to corotation) and damping (due to infall)

==> **Net result:**

In standard thin disk, p-modes are likely damped
==> No HFQPOs in thermal state

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Real disks (in "transition state") are more complicated…

Magnetic fields…

Effects of Magnetic Fields on P-Modes

-- Mode frequencies are slightly/modestly affected: Robust…

-- Mode growth rates can be significantly affected:

1. Magnetic fields may accumulate inside r_{isco}
(e.g. Lovelace et al 2009; simulations by Hawley, Krolik etc)

==> The inner disk edge may be more reflective than
standard SS disk (Tsang & DL 2009b)

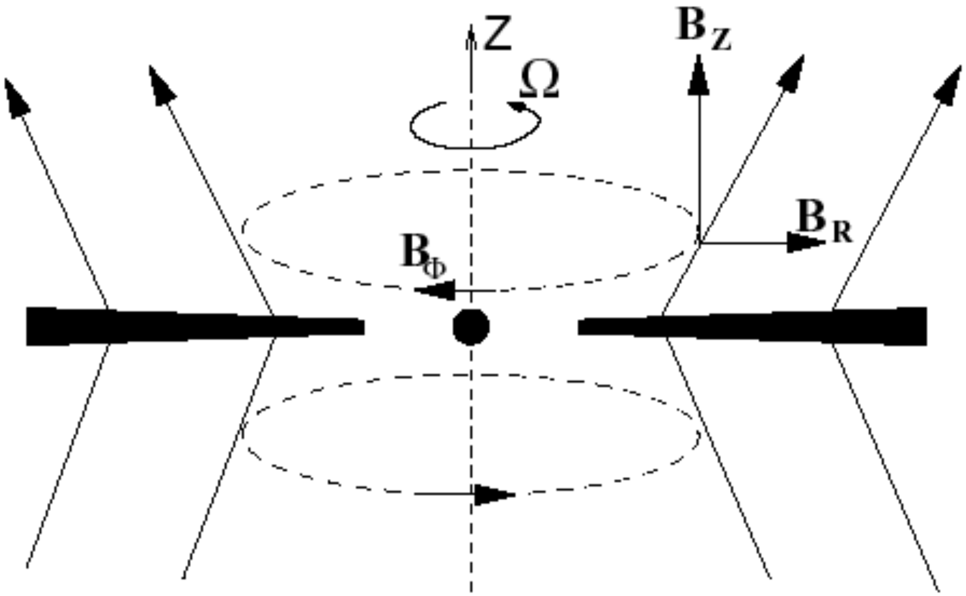
==> Enhance net growth of p-modes

2. Large-scale poloidal field can enhance corotational instability

==> Enhance growth of p-modes

(Fu & DL 2012; cf. AEI by Tagger & Pallet 1999; Tagger & Varniere 2006)

Disks threaded by large-scale poloidal magnetic fields (embedded in a corona)



-- Increase the p-mode growth rate due to corotation resonance

Disk + Corona (coupled by B field) oscillate together,
the “clock” is mainly set by disk

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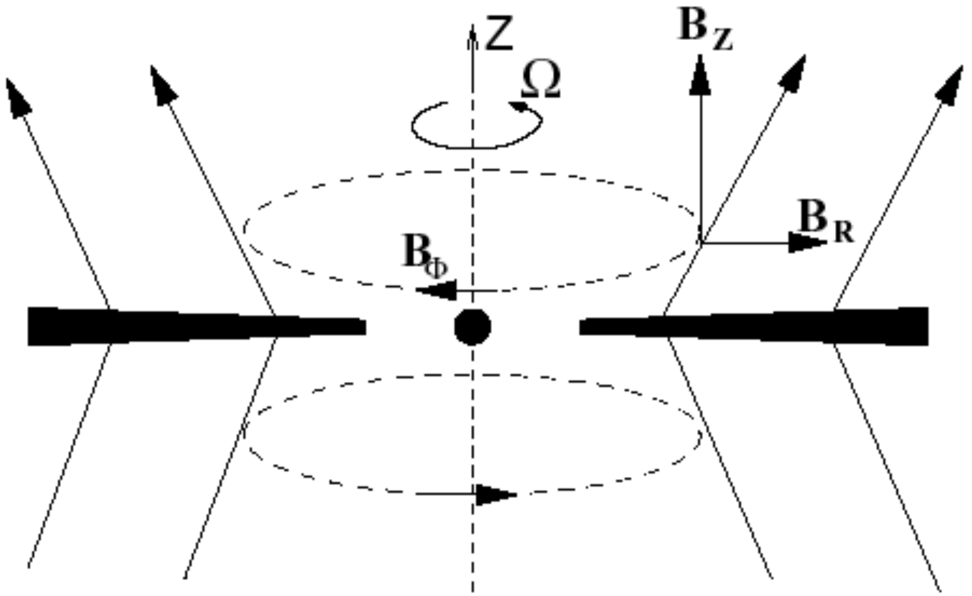

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-- Such large-scale field is ideal for producing jets/outflows

So far everything I have said can be backed up by concrete calculations (at least for idealized setup)…

What about **REAL** accretion flows in “Transitional State”?

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What about **REAL** accretion flows in “Transitional State”?

We suggest:

In the transitional state, large-scale B fields are temporarily/episodically created (e.g., fields buoyantly rise from collapsed ADAF and reconnect)

==> (1) Allow p-modes to grow ==> HFQPOs

(2) Produce episodic jets

Summary

- There are not many viable theoretical models for HFQPOs

- P-modes (spiral density modes) partially trapped in the inner-most region of disks is promising candidate:

  -- Frequencies can be calculated from first principle

  $$\omega \simeq (0.55-0.75)\, m\, \Omega(r_{\rm in})$$

  robust, agree with observations (consistent mass, spin)

  -- Can grow naturally due to corotation resonance (GR important)

  -- In this theory, there are good reasons to expect that

  (i) They are not present in thermal state (mode damping due to infall)

  (ii) They are present in transitional state (effect of large-scale B field)

- Incomplete: Other issues/complications; analytic+simulations important…

How to kill my theory for HFQPOs?

Show that in the transitional state, the disk is truncated at $r_{in} > 2\ r_{isco}$

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Thanks!